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BAYESIAN ESTIMATION OF RELIABILITY IN TWO-PARAMETER GEOMETRIC DISTRIBUTION

Sudhansu S. Maiti [1]* **and Sudhir Murmu** [2]
[1] Department of Statistics, Visva-Bharati University, Santiniketan, India
[2] District Rural Development Agency, Khunti, Jharkand, India
E Mail: [1]* dssm1@rediffmail.com, [2] sudhir.murmu@yahoo.com

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Abstract

Bayesian estimation of reliability of a component, $R(t) = P(X \geq t)$, when X follows two-parameter geometric distribution, has been considered. Maximum Likelihood Estimator (MLE), an Unbiased Estimator and Bayesian Estimator have been compared. Bayesian estimation of component reliability $R = P(X \leq Y)$, arising under stress-strength setup, when Y is assumed to follow independent two-parameter geometric distribution has also been discussed assuming independent priors for parameters under different loss functions.

Key Words: ML Estimator, Quasi-Bayes Estimate, Unbiased Estimator.

1. Introduction

Various lifetime models have been proposed to describe the important characteristics of lifetime data. Most of these models assume lifetime to be a continuous random variable. However, it is sometimes impossible or inconvenient to measure the life length of a device on a continuous scale. In practice, we come across situations where lifetimes are recorded on a discrete scale. Discrete life distributions have been suggested and properties have been studied by Barlow and Proschan [1]. Here one may consider lifetime to be the number of successful cycles or operations of a device before failure. For example, the bulb in Xerox machine lights up each time a copy is taken. A spring may breakdown after completing a certain number of cycles of 'to-and-fro' movements.

The study of discrete distributions in lifetime models is not very old. Yakub and Khan [11] considered the geometric distribution as a failure law in life testing and obtained various parametric and nonparametric estimation procedures for reliability characteristics. Bhattacharya and Kumar [2] have considered the parametric as well as Bayesian approach to the estimation of the mean life cycle and that of reliability function for complete as well censored sample. Krishna and Jain [3] obtained classical and Bayes estimation of reliability for some basic system configurations.

The geometric distribution [abbreviated as $Geo(\theta)$] is given by

$$P(X = x) = (1-\theta)\theta^{x}; x = 0, 1, 2, \ldots\ldots; \quad 0 < \theta < 1$$

and the component reliability is given by

$$R(t) = \theta^t;\ t = 0, 1, 2, \ldots\ldots\ldots \quad (1)$$

Modeling in terms of two-parameter geometric and estimation of its parameters and related functions are of special interest to a manufacturer who wishes to offer a minimum warranty life cycle of the items produced.

The two-parameter geometric distribution abbreviated as $Geo(r, \theta)$] is given by

$P(X = x) = (1-\theta)\theta^{x-r};\ x = r, r+1, r+2, \ldots.\ 0 < \theta < 1$ and $r \in \{0, 1, 2, \ldots..\}$
and the component reliability is given by

$$R(t) = \theta^{t-r};\ t = r, r+1, r+2, \ldots\ldots\ldots \quad (2)$$

The continuous counterpart of geometric (i.e. exponential) distribution, one-parameter as well as two-parameter is considered by a host of authors (see ref. Sinha [10]).

In the stress-strength setup, $R = P(X \le Y)$ originated in the context of the reliability of a component of strength Y subjected to a stress X. The component fails if at any time the applied stress is greater than its strength and there is no failure when $X \le Y$. Thus R is a measure of the reliability of the component. Many authors considered the problem of estimation of R in continuous setup in the past. In the discrete setup, a limited work has been done so far. Maiti [4] has considered stress (or demand) X and strength (or supply) Y as independently distributed geometric random variables, whereas Sathe and Dixit [9] assumed as negative binomial variables, and derived both MLE and UMVUE of R. Maiti and Kanji [7] has derived some expressions of R using a characterization of $P(X \le Y)$ and Maiti ([5], [6]) considered MLE, UMVUE and Bayes Estimation of R for some discrete distributions useful in life testing.

If X and Y follow two-parameter geometric distributions with parameters (θ_1, r_1) and (θ_2, r_2) respectively, then

$$\begin{aligned} R &= \rho\theta_2^{\delta} \text{ for } \delta > 0 \\ &= 1-(1-\rho)\theta_1^{-\delta} \text{ for } \delta < 0, \end{aligned} \quad (3)$$

where $\rho = \dfrac{1-\theta_1}{1-\theta_1\theta_2}$ and $\delta = r_1 - r_2$.

The objective of this article is to compare the estimates of reliability for mission time as well as stress-strength set up for two-parameter geometric distribution. The paper is organized as follows. In section 2, we have summarized MLE, Unbiased Estimator (c.f. Maiti et al. [8]) and derived Quasi-Bayes estimate of $R(t)$ under different loss functions assuming conjugate priors for the parameters involved. We attempt Quasi-Bayes estimate since derivation of posterior distribution seems to be intractable. Different scale invariant loss functions are considered, viz., squared error loss, squared log error loss, Modified Linear Exponential (MLINEX) loss, Absolute error loss and the corresponding estimates have been compared. We have discussed MLE, Unbiased Estimator and Quasi-Bayes estimates of R in section 3. All comparisons have been made through simulation study in section 4. Section 5 concludes.

2. Inference on $R(t)$

Let $(X_1, X_2, ..., X_n)$ be a random sample from $Geo(r, \theta)$. Maximum Likelihood Estimator of r and θ are $X_{(1)}$ and $\frac{S}{n+S}$ respectively, where $X_{(1)}$ is smallest observation among $X_1, X_2, ..., X_n$ and $S = \sum_{i=1}^{n}(X_i - X_{(1)})$. ML Estimators of $R(t)$ is given by

$$\hat{R}_M(t) = 1 \qquad \text{for } t \le X_{(1)}$$

$$= \left[\frac{S}{n+S}\right]^{t - X_{(1)}} \quad \text{for } t > X_{(1)}.$$

Here $(X_{(1)}, S)$ is sufficient statistic for (r, θ), but it is not complete (c.f. Maiti et al. [8]).

For $n = 1$,

$$f(x \mid X_{(1)}, S) = 1$$

For $n = 2$

$$f(x \mid X_{(1)}, S) = \frac{1}{2} \quad \text{if } x = X_{(1)}$$

$$= \frac{1}{2} \quad \text{if } x = X_{(1)} + S$$

For $n \ge 3$, $S < n$,

$$f(x \mid X_{(1)}, S) = \binom{S - (x - X_{(1)}) + n - 2}{S - (x - X_{(1)})} \Big/ \binom{S + n - 1}{S} \quad \text{if } X_{(1)} \le x \le X_{(1)} + S$$

$$= 0 \qquad \text{otherwise}$$

For $n \ge 3, S \ge n$,

$$f(x \mid X_{(1)}, S) = \binom{S + n - 2}{S} \Big/ \left\{\binom{S + n - 1}{S}\right\} - \binom{S - 1}{n - 1} \quad \text{if } x = X_{(1)}.$$

$$= \left\{ \binom{S-\left(x-X_{(1)}+n-2\right)}{S-\left(x-X_{(1)}\right)} - \binom{S-\left(x-X_{(1)}\right)-1}{n-2} \right\} \Big/ \left\{ \binom{S+n-1}{S} - \binom{S-1}{n-1} \right\}$$

if $X_{(1)} < x \le X_{(1)} + S - (n-1)$

$$= \binom{S-\left(x-X_{(1)}+n-2\right)}{S-\left(x-X_{(1)}\right)} \Big/ \left\{ \binom{S+n-1}{S} - \binom{S-1}{n-1} \right\}$$

if $X_{(1)} + S - (n-1) < x \le X_{(1)} + S$

= 0 Otherwise.

Hence, using the Rao-Blackwell theorem, an unbiased estimator of $R(t)$ is given as follows:

For $n = 1$,

$$\hat{R}_U(t) = 1 \quad \text{if } t \le X_{(1)}$$
$$= 0 \quad \text{if } t > X_{(1)}.$$

For $n = 2$,

$$\hat{R}_U(t) = 1 \quad \text{if } t \le X_{(1)}$$
$$= \frac{1}{2} \quad \text{if } X_{(1)} < t \le X_{(1)} + S$$
$$= 0 \quad \text{if } t > X_{(1)}. + S.$$

For $n \ge 3$ and $S < n$,

$$\hat{R}_U(t) = 1 \quad \text{if } t \le X_{(1)}$$

$$= \sum_{x=t}^{X_{(1)}+S} \frac{n-1}{X_{(1)}+S+n-1} \prod_{j=1}^{n-2} \frac{\left(X_{(1)}+S+n-x1-j\right)}{\left(X_{(1)}+S+n-1-j\right)}$$

if $X_{(1)} < t \le X_{(1)} + S$

= 0 if $t > X_{(1)}. + S.$

For $n \ge 3$ and $S \ge n$,

$$\hat{R}_U(t) = 1 \quad \text{if } t \le X_{(1)}$$

$$= \sum_{x=t}^{X_{(1)}+S-(n-1)} \left\{ \binom{S-\left(x-X_{(1)}\right)+n-2}{S-\left(x-X_{(1)}\right)} - \binom{S-\left(x-X_{(1)}\right)-1}{n-2} \right\} \Big/ \left\{ \binom{S+n-1}{S} - \binom{S-1}{n-1} \right\}$$

$$+\sum_{x=X_{(1)}-(n-1)+1}^{X_{(1)}+S}\binom{S-(x-X_{(1)}+n-2)}{S-(x-X_{(1)})}\Bigg/\left\{\binom{S+n-1}{S}-\binom{S-1}{n-1}\right\}$$

if $X_{(1)} < t \le X_{(1)} + S - (n-1)$

$$=\sum_{x=t}^{X_{(1)}+S}\binom{S-(x-X_{(1)}+n-2)}{S-(x-X_{(1)})}\Bigg/\left\{\binom{S+n-1}{S}-\binom{S-1}{n-1}\right\}$$

if $X_{(1)} + S - (n-1) < t \le X_{(1)} + S$

$= 0$ Otherwise.

We obtain the Bayes estimates of the parameters under the assumptions that the parameters θ and r are random variables. It is assumed that θ and r have independent beta and Poisson priors as follows:

$$\pi(\theta)=\frac{1}{B(p,q)}\theta^{p-1}(1-\theta)^{q-1}; \quad 0<\theta<1,\ p,q>0$$

and

$$\phi(r)=e^{-\lambda}\frac{\lambda^{r}}{r!}; \quad r = 0, 1, 2\ldots$$

Here $B(p,q)=\int_0^1 \theta^{p-1}(1-\theta)^{q-1}\,d\theta.$

The joint distribution of θ and r given observations is given by

$$g(\theta,r\mid X)=L(x\mid r,\theta).\pi(r,\theta)$$

$$=(1-\theta)^{n}\,\theta^{\sum_{i=1}^{n}(x_i-r)}.\frac{1}{B(p,q)}\theta^{p-1}(1-\theta)^{q-1}.e^{-\lambda}\frac{\lambda^{r}}{r!}$$

$$g(\theta,r\mid X)\propto \theta^{S+n(X_{(1)}-r)+p-1}(1-\theta)^{n+q-1}e^{-\lambda}\frac{\lambda^{r}}{r!};\quad 0<\theta<1,$$

$$r = 0,1,2,\ldots\ldots X_{(1)}.$$

The posterior distributions of θ and r are as follows:

$$g_1(r\mid X_{(1)},S)=\frac{e^{-\lambda}\frac{\lambda^{r}}{r!}B\big(S+n(X_{(1)}-r)+p,n+q\big)}{\sum_{w=0}^{X_{(1)}}e^{-\lambda}\frac{\lambda^{w}}{w!}B\big(S+n(X_{(1)}-w)+p,n+q\big)}$$

and

$$g_2\left(\theta \mid X_{(1)}, S\right) = \theta^{S+nX_{(1)}+p-1}\left(1-\theta\right)^{n+q-1} \times$$

$$= \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \frac{\left(\lambda/\theta^n\right)^w}{w!}}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \frac{\lambda^w}{w!} B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

Derivation of posterior distribution of $R(t)$ seems to be intractable. Therefore, Quasi-Bayes estimates of these expressions by substituting Bayes estimates of the parameters involved have been obtained. Comparisons have been made through simulation study. Some scale invariant loss functions have been considered to get Bayes estimates of θ and r. These are summarized in the following discussion.

2.1 Squared Error Loss

The loss function is given by

$$L_1(\alpha,\delta) = \left(1-\frac{\delta}{\alpha}\right)^2$$

and estimates of θ and r are

$$\hat{\theta} = E\left(\frac{1}{\theta^2}.\theta \mid X_{(1)}, S\right) = \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \frac{\lambda^w}{w!} B\left(S+n\left(X_{(1)}-w\right)+p-1, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \frac{\lambda^w}{w!} B\left(S+n\left(X_{(1)}-w\right)+p-2, n+q\right)}$$

and

$$\hat{r} = E\left(\frac{1}{r^2} r \mid X_{(1)}, S\right) = \frac{\sum_{w=1}^{X_{(1)}} \frac{1}{w} e^{-\lambda} \frac{\lambda^w}{w!} B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}{\sum_{w=1}^{X_{(1)}} \frac{1}{w^2} e^{-\lambda} \frac{\lambda^w}{w!} B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

respectively.

2.2. Squared log error loss

The loss function is given by

$$L_2(\alpha,\delta) = \left(\ln\delta - \ln\alpha\right)^2 = \left(\ln\frac{\delta}{\alpha}\right)^2$$

and estimates of θ and r are

$$\hat{\theta} = e^{E\left(\ln\theta | X_{(1)}, S\right)},$$

where

$$E\left(\ln\theta \mid X_{(1)}, S\right) = \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}\int_0^1 \ln\theta.\theta^{S+nX_{(1)}+p-1}(1-\theta)^{n+q-1}d\theta}{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

$$= \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}.B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)\times I(w)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

where $I(w) = \left[\psi\left(S+n\left(X_{(1)}-w\right)+p\right)-\psi\left(S+n\left(X_{(1)}-w\right)+p+q+n\right)\right]$

and $\psi(u)$ is digamma function.

and

$$\hat{r} = e^{E\left(\ln r | X_{(1)}, S\right)},$$

where

$$E\left(\ln r \mid X_{(1)}, S\right) = \frac{\sum_{w=1}^{X_{(1)}} \ln w.e^{-\lambda}\frac{\lambda^w}{w!}.B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

2.3 MLINEX

The loss function is given by

$$L_3(\alpha,\delta) = c\left[\left(\frac{\delta}{\alpha}\right)^{\gamma} - \gamma\ln\frac{\delta}{\alpha} - 1\right]; \gamma \neq 0, c > 0$$

and estimates of θ and r [assuming $c = 1$] are

$\hat{\theta} = \left[E\left(\theta^{-\gamma} \mid X_{(1)}, S\right)\right]^{-\frac{1}{\gamma}}$, where

$$E\left(\theta^{-\gamma} \mid X_{(1)}, S\right) = \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}B\left(S+n\left(X_{(1)}-w\right)+p-\gamma, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda}\frac{\lambda^w}{w!}B\left(S+n\left(X_{(1)}-w\right)+p, n+q\right)}$$

and $\hat{r} = \left[E\left(r^{-\gamma} \mid X_{(1)}, S\right)\right]^{-\frac{1}{\gamma}}$,

where $E\left(r^{-\gamma} \mid X_{(1)}, S\right) = \dfrac{\sum_{w=0}^{X_{(1)}} w^{-\gamma}.e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)}$

Particular Case: when $\gamma = 1$, we have Entropy loss function. Then

$$\hat{\theta} = \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p - 1, n+q\right)}$$

and

$$\hat{r} = \frac{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p - 2, n+q\right)}$$

2.4 Absolute error loss

The loss function is given by

$$L_4(\alpha, \delta) = \left|1 - \frac{\delta}{\alpha}\right|$$

Then $\hat{\theta} = M$ such that

$$\frac{\int_0^M \frac{1}{\theta} g_2\left(\theta \mid X_{(1)}, S\right) d\theta}{\int_0^1 \frac{1}{\theta} g_2\left(\theta \mid X_{(1)}, S\right) d\theta} = \frac{1}{2}$$

i.e. $\dfrac{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} \int_0^M \theta^{S + n\left(X_{(1)} - r\right) + p - 2} (1-\theta)^{n+q-1} d\theta}{\sum_{w=0}^{X_{(1)}} e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p - 1, n+q\right)} = \dfrac{1}{2}$

To get the Bayes estimate of r, we have to solve the following equation for M.

$$\frac{\sum_{w=1}^{M)} \frac{1}{w}.e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)}{\sum_{w=1}^{X_{(1)}} \frac{1}{w}.e^{-\lambda} \dfrac{\lambda^w}{w!} B\left(S + n\left(X_{(1)} - w\right) + p, n+q\right)} = \frac{1}{2}.$$

3. Inference on R

Let $\left(X_1, X_2, \ldots\ldots X_{n_1}\right)$ and $\left(Y_1, Y_2, \ldots\ldots, Y_{n_2}\right)$ be random samples from $Geo(r_1, \theta_1)$ and $Geo(r_2, \theta_2)$ respectively. $\left(X_{(1)}, S_1\right)$ and $\left(Y_{(1)}, S_2\right)$ are defined in the same way as in section 2. Hence ML Estimator of R is given by

$$\hat{R}_M = \hat{\rho}\left(\frac{S_2}{n_2 + S_2}\right)^{\hat{\delta}} \qquad \text{for } \hat{\delta} > 0$$

$$= 1 - (1 - \hat{\rho})\left(\frac{S_1}{n_1 + S_1}\right)^{-\hat{\delta}} \qquad \text{for } \hat{\delta} < 0,$$

where $\hat{\rho} = \dfrac{n_1 n_2 + n_1 S_2}{n_1 n_2 + n_1 S_2 + n_2 S_1}$ and $\hat{\delta} = X_{(1)} - Y_{(1)}$.

Application of the Rao-Blackwell theorem gives an unbiased estimator of R as

$$\hat{R}_U = \frac{1}{n_1} + \sum_{x=X_{(1)}+1}^{Y_{(1)}} f\left(x \mid X_{(1)}, S_1\right) + \sum_{x=Y_{(1)}}^{\min(W_1, W_2)} \sum_{y=x}^{W_2} f\left(x \mid X_{(1)}, S_1\right) f\left(y \mid Y_{(1)}, S_2\right) \qquad \text{if } X_{(1)} < Y_{(1)}$$

$$= \frac{1}{n_1} + \sum_{x=Y_{(1)}}^{\min(W_1, W_2)} \sum_{y=x}^{W_2} f\left(x \mid X_{(1)}, S_1\right) f\left(y \mid Y_{(1)}, S_2\right) \qquad \text{if } X_{(1)} = Y_{(1)}$$

$$= \frac{1}{n_1} + \sum_{y=X_{(1)}}^{W_2} f\left(y \mid Y_{(1)}, S_2\right) + \sum_{x=X_{(1)}+1}^{\min(W_1, W_2)} \sum_{y=x}^{W_2} f\left(x \mid X_{(1)}, S_1\right) f\left(y \mid Y_{(1)}, S_2\right) \qquad \text{if } X_{(1)} > Y_{(1)}$$

where, $W_1 = X_{(1)} + S_1$, $W_2 = Y_{(1)} + S_2$. The variance of this unbiased estimator will be smaller than the unbiased estimator $\frac{1}{n_1 n_2} \sum_{i=1}^{n_1} \sum_{j=1}^{n_2} I\left(X_i < Y_j\right)$. Derivation of posterior distribution of R in this case seems to be intractable. Therefore, Quasi-Bayes estimates of R by substituting Bayes estimates of the parameters derived in section 2 have been found out.

4. Simulation and Discussion

We generate sample of size n and on the basis of this sample, calculate MLE and UE and Quasi-Bayes estimate of $R(t)$ and their Mean Squared Errors (MSEs). 10000 such estimates have been calculated and results, on the basis of these estimates have been reported in Tables 1-4. In each table, there are six rows in average estimate of reliability and their MSEs. Information reported as 1st row: MLE, 2nd row: UE, 3rd row: Bayes estimate under squared error loss, 4th row: Bayes estimate under squared log error, 5th row: Bayes estimate under entropy loss, 6th row: Bayes estimate under Absolute error loss. Each table has been prepared considering different choices of a particular parameter, keeping others fixed at initial set up. All simulations and calculations have been done using R-Software and algorithms used can be obtained by contacting the corresponding author.

Maiti et al. [8] have reported that MLE of $R(t)$ perform better in mean square error sense if $0.02 < R(t) < 0.5$; otherwise UE performs well. From tables 1-4, it is observed that quasi-Bayes estimates are better than MLE as well as UE in all most all cases. Among four quasi-Bayes estimates, the estimate under squared error loss seems better, whereas the estimate under absolute error, the performance is not encouraging.

We also generate samples of size n_1 and n_2, and on the basis of these samples, calculate MLE and UE and Quasi-Bayes estimate of R and their Mean Squared Errors (MSEs). Here, we take $n_1 = n_2 = 10,$ and 1000 estimates of R have been taken for calculating MSEs [Tables 5-6]. Maiti et al. [8] have reported that UE of R perform better in mean square error sense for extreme low and high reliable components; in other cases, MLE is better. But in all most all case, quasi-Bayes estimates are better.

5. Concluding Remark

This paper takes into account the Bayes estimation aspect of reliability with two-parameter geometric lifetime. The continuous distributions are widely referenced probability laws used in reliability and life testing for continuous data. When the lives of some equipment and components are being measured by the number of completed cycles of operations or strokes, or in case of periodic monitoring of continuous data, the discrete distribution is a natural choice. Bayesian Estimation procedures have been worked out for estimating reliability assuming independent priors for parameters under different loss functions. In all most all cases, quasi-Bayes estimates of reliability are better in mean square error sense. Some other distributions that are used as discrete life distributions are to be considered and their Bayes estimation aspect of reliability are to be attempted in future.

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t	Reliability	Avg. Estimates	MSE
20	0.32768	**0.3166573**	**0.006389343**
		0.324255	**0.00666498**
		0.3196006	0.005297303
		0.3256051	0.005139682
		0.3236131	0.5139682
		0.3328666	0.005644666
25	0.1073742	**0.1090191**	**0.002340292**
		0.1079153	**0.002589489**
		0.1035126	0.002289946
		0.1071390	0.002308059
		0.1059272	0.002299524
		0.1111683	0.002561238
30	0.03518437	**0.03964109**	**0.0007162865**
		0.03618544	**0.0007241272**
		0.03638538	0.0005862288
		0.03813851	0.0006180215
		0.03754956	0.0006067526
		0.04018129	0.0007150897
35	0.0115292	**0.01482919**	**0.000193777**
		0.01198529	**0.0001630661**
		0.01269303	0.0001348279
		0.01346045	0.0001460612
		0.01320140	0.0001421481
		0.01440264	0.0001740828

Table 1: Average Estimates and Mean Square Errors of $R(t)$ with n=20, r=15, p=8, q=2.

n	Reliability	Avg. Estimates	MSE
10	0.1073742	**0.1060938**	**0.00479161**
		0.1265446	**0.007624039**
		0.09549643	0.003795114
		0.1042998	0.003998701
		0.1013891	0.003922511
		0.1330134	0.009238643
15	0.1073742	**0.1084599**	**0.003447145**
		0.1217706	**0.004772602**
		0.1045881	0.003014518
		0.1098457	0.003102323
		0.1080907	0.003068467
		0.1185306	0.004424606
20	0.1073742	**0.1086262**	**0.002678777**
		0.1184946	**0.003431893**
		0.1041115	0.002489043
		0.1079099	0.002514939
		0.1066405	0.002503606
		0.1121600	0.002841681
25	0.1073742	**0.1080681**	**0.002088382**
		0.1158652	**0.002549381**
		0.1050123	0.001967833
		0.1080312	0.001982178
		0.1070225	0.001975586
		0.1114314	0.002153514

Table 2: Average Estimates and Mean Square Errors of $R(t)$ with r=15, t=25, p=8, q=2.

r	Reliability	Avg. Estimates	MSE
5	0.01152922	**0.0146542**	**0.000238691**
		0.01405295	**0.0002928919**
		0.01356151	0.0001464932
		0.01438816	0.0001601152
		0.01410971	0.00015539861
		0.01536586	0.0001893861
10	0.03518437	**0.0384844**	**0.000921849**
		0.04098456	**0.001267719**
		0.03600980	0.0006123444
		0.03781796	0.0006448308
		0.03721048	0.0006333065
		0.03980784	0.0007365458
15	0.1073742	**0.1086646**	**0.003386037**
		0.1220295	**0.004699975**
		0.105009	0.002266010
		0.1088256	0.002296038
		0.107502	0.002283241
		0.1130798	0.002605971
20	0.32768	**0.3177951**	**0.008842791**
		0.35122351	**0.01040297**
		0.3101675	0.006174129
		0.3165970	0.005882643
		0.3144629	0.005970977
		0.3250680	0.006599251

Table 3: Average Estimates and Mean Square Errors of $R(t)$ with n=20, t=25, p=8, q=2.

θ	Reliability	Avg. Estimates	MSE
0.7	0.02824752	**0.03177319**	**0.000551296**
		0.03256912	**0.0006866504**
		0.02770768	0.000340732
		0.02991801	0.0003669113
	p=7, q=3	0.02917032	0.0003570654
		0.03059155	0.000396458
0.8	0.1073742	**0.1069406**	**0.002548496**
		0.1166341	**0.003241387**
		0.1060890	0.002650470
		0.1098881	0.02693512
	p=8, q=2	0.1086186	0.002676484
		0.1142395	0.003059418
0.9	0.3486784	**0.3413777**	**0.006712354**
		0.3679966	**0.007528894**
		0.3315982	0.006204844
		0.3363357	0.006150177
	p=9, q=1	0.3347966	0.006167038
		0.3841432	0.01139207
0.93	0.4839823	**0.4763589**	**0.006831021**
		0.5050617	**0.007228364**
		0.4655819	0.005150811
		0.4723362	0.005180253
	P=13, q=1	0.4702358	0.005167441
		0.5644204	0.1685316
0.96	0.6648326	**0.6677073**	**0.005206639**
		0.6918656	**0.005490035**
		0.6256307	0.004290399
		0.6394016	0.003793488
	p=24, q=1	0.6353227	0.00390119
		0.953012	0.1613833
0.99	0.9043382	**0.9352147**	**0.00497277**
		0.9412775	**0.002639834**
		0.8405322	0.004131144
	p=99, q=1	0.8572153	0.002276667
		0.8522604	0.002768958
		0.980264	0.230441

Table 4: Average Estimates and Mean Square Errors of $R(t)$ with n=20, r=15, t=25.

$r_2 \mid r_1$	5			10		
	Reliability	Avg. Estimates	MSE	Reliability	Avg. Estimates	MSE
5	0.588235	**0.593202**	**0.013734**	0.098864	**0.103300**	**0.004158**
		0.589705	**0.013978**		**0.100187**	**0.004813**
		0.593160	0.008214		0.088356	0.002387
		0.589218	0.007796		0.094898	0.002323
		0.590506	0.007942		0.092725	0.002338
		0.589435	0.008925		0.094697	0.002533
10	0.930794	**0.928161**	**0.002695**	0.588235	**0.593680**	**0.012493**
		0.933117	**0.003082**		**0.590026**	**0.012775**
		0.935221	0.001711		0.584252	0.008245
		0.929792	0.001753		0.580510	0.007879
		0.931604	0.001735		0.581726	0.008002
		0.929551	0.001922		0.580201	0.009766
15	0.988368	**0.984090**	**0.000344**	0.930794	**0.925457**	**0.002877**
		0.988884	**0.000283**		**0.930138**	**0.003267**
		0.987664	0.000160		0.932667	0.001880
		0.985964	0.000186		0.927189	0.001946
		0.986543	0.000176		0.929023	0.001919
		0.985866	0.000205		0.927168	0.002199
20	0.998045	**0.995580**	$\mathbf{6.22} \times 10^{-5}$	0.988368	**0.983920**	**0.000375**
		0.997830	$\mathbf{3.44} \times 10^{-5}$		**0.988676**	**0.000319**
		0.997158	2.39×10^{-5}		0.987053	0.000177
		0.996679	2.90×10^{-5}		0.985314	0.000207
		0.996845	2.72×10^{-5}		0.985909	0.000196
		0.996672	3.18×10^{-5}		0.985451	0.000232

Table 5a: Average Estimates and Mean Square Errors of R with
$n_1 = n_2 = 10, p_1 = 7, q_1 = 3, p_2 = 7, q_2 = 3$

$r_2 \mid r_1$	15			20		
	Reliability	Avg. Estimates	MSE	Reliability	Avg. Estimates	MSE
5	0.016616	**0.021989**	**0.000582**	0.002792	**0.005531**	**8.08** $\times 10^{-5}$
		0.016748	**0.000526**		**0.002837**	**4.82** $\times 10^{-5}$
		0.017008	0.000286		0.003660	2.44×10^{-5}
		0.019145	0.000324		0.004284	3.04×10^{-5}
		0.184193	0.000310		0.004069	2.83×10^{-5}
		0.019219	0.000355		0.004237	3.30×10^{-5}
10	0.098864	**0.098864**	**0.004131**	0.001661	**0.019886**	**0.000460**
		0.095907	**0.004796**		**0.014591**	**0.000423**
		0.095491	0.002746		0.016505	0.000241
		0.102057	0.002773		0.018651	0.0002736
		0.099870	0.002757		0.017918	0.000261
		0.101985	0.003164		0.018323	0.000297
15	0.588235	**0.584511**	**0.012804**	0.098864	**0.102787**	**0.004199**
		0.580772	**0.013095**		**0.099599**	**0.004870**
		0.593470	0.006776		0.095308	0.002635
		0.589606	0.006541		0.101879	0.002674
		0.590869	0.006622		0.099684	0.002654
		0.589494	0.011931		0.100602	0.003813
20	0.930794	**0.925069**	**0.002907**	0.588235	**0.588104**	**0.013467**
		0.929745	**0.003305**		**0.584571**	**0.013697**
		0.930670	0.001782		0.589184	0.006215
		0.925033	0.001879		0.585346	0.006211
		0.926922	0.001842		0.586599	0.006222
		0.924809	0.002638		0.578639	0.020662

Table 5b: Average Estimates and Mean Square Errors of R with
$n_1 = n_2 = 10, p_1 = 7, q_1 = 3, p_2 = 7, q_2 = 3$

$r_2 \mid r_1$	5			10		
	Reliability	Avg. Estimates	MSE	Reliability	Avg. Estimates	MSE
5	0.526315	**0.527212**	**0.017147**	0.310478	**0.301431**	**0.012172**
		0.526890	**0.015018**		**0.310643**	**0.011810**
		0.527249	0.012307		0.309978	0.008902
		0.526866	0.0126550		0.310687	0.009150
		0.526997	0.012590		0.310478	0.009102
		0.525530	0.015986		0.274773	0.013459
10	0.720294	**0.728055**	**0.011734**	0.526315	**0.517795**	**0.015961**
		0.718481	**0.011729**		**0.517974**	**0.014000**
		0.721179	0.008835		0.524161	0.009955
		0.719954	0.009045		0.523790	0.010837
		0.720326	0.009006		0.523915	0.010609
		0.749458	0.012557		0.519945	0.024380
15	0.834836	**0.845243**	**0.007065**	0.720294	**0.725654**	**0.012068**
		0.838176	**0.007881**		**0.715987**	**0.012066**
		0.826150	0.005976		0.708335	0.007628
		0.827784	0.0060043		0.7122060	0.008112
		0.827390	0.0060046		0.711245	0.007982
		0.870079	0.007777		0.756649	0.018033
20	0.902472	**0.909896**	**0.003701**	0.834836	**0.842101**	**0.007403**
		0.907375	**0.004407**		**0.834737**	**0.007828**
		0.876734	0.005034		0.793630	0.006973
		0.884843	0.004444		0.808061	0.006014
		0.882684	0.004592		0.804243	0.006224
		0.931743	0.004333		0.873130	0.009625

Table 6a: Average Estimates and Mean Square Errors of R with
$n_1 = n_2 = 10, p_1 = 9, q_1 = 1, p_2 = 9, q_2 = 1$

$r_2 \mid r_1$	15			20		
	Reliability	Avg. Estimates	MSE	Reliability	Avg. Estimates	MSE
5	0.183515	**0.176727**	**0.007721**	0.108363	**0.104030**	**0.004384**
		0.185015	**0.008525**		**0.107695**	**0.005208**
		0.192788	0.006518		0.134013	0.004755
		0.190298	0.006536		0.124208	0.004076
		0.190934	0.006540		0.126825	0.004242
		0.140342	0.009060		0.068923	0.004552
10	0.310784	**0.299440**	**0.012136**	0.183515	**0.182073**	**0.008720**
		0.308754	**0.011652**		**0.190438**	**0.009657**
		0.323621	0.008177		0.230966	0.008483
		0.318944	0.008760		0.214370	0.007319
		0.320141	0.008602		0.218775	0.007569
		0.269985	0.020795		0.142069	0.012357
15	0.526315	**0.528051**	**0.016868**	0.310784	**0.302399**	**0.012456**
		0.527508	**0.014852**		**0.311517**	**0.012054**
		0.523521	0.005220		0.371186	0.008077
		0.523198	0.007006		0.350055	0.007246
		0.523318	0.006485		0.355758	0.007328
		0.516243	0.038350		0.285457	0.031339
20	0.720294	**0.728946**	**0.012284**	0.526315	**0.531249**	**0.015239**
		0.719389	**0.012217**		**0.530590**	**0.013301**
		0.663570	0.008039		0.521966	0.001070
		0682566	0.007419		0.522492	0.002797
		0.677480	0.007469		0.522351	0.002149
		0.740730	0.028737		0.518026	0.055713

Table 7b: Average Estimates and Mean Square Errors of R with
$n_1 = n_2 = 10, p_1 = 9, q_1 = 1, p_2 = 9, q_2 = 1$