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Local Stability and Structure of a Differentially Rotating Star of Non-Uniform Density

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Abstract
A method is proposed to compute the theoretical estimation of physical parameters and stability of differential rotation for polytropic stars including mass variation. The law of differential rotation is assumed to be in the form $\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4$, the angular velocity of rotation (ω) is a function of distance (s) of the fluid element from the axis of rotation. Utilizing the concepts of Roche- equipotential and averaging approach of (Kippenhahn and Thomas, 1970) in a manner, earlier used by (Saini, et al., 2012) to incorporate the effects of differential rotation on the equilibrium structure of polytropic stellar models. The inner structure of differentially rotating polytropic models of a star is demonstrated by calculating various physical parameters for suitable combinations of parameters.

Keywords- Roche-Equipotential, Equilibrium Structure, Tidal Distortion, Differential Rotation, Mass Variation.

1. Introduction

Almost each known star rotates about its axis and it is also observed that the rotation may be a solid body rotation or differential rotation. In case of binary star system, primary component (more massive) generally remains larger in comparison to its secondary component. Most stars of binary systems are rotating uniformly and revolving around their common centre of mass. It is expected that some of the stars in binary system rotates differentially about their axis. Differential rotations influence the inner structures and equilibrium configurations of such stars. It is also expected that the equilibrium structure of a star in binary system is also influenced by the combined effects of differential rotation and tidal forces.

The physical parameters related to the structures of differentially rotating gaseous spheres are calculated using the law of differential rotation from $\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4$, where $\omega(s)$ is angular velocity of rotation of a fluid element at distance s from the axis of rotation and b_1, b_2 and b_3 are numerical constants. Our technique utilizes the averaging approach of (Kippenhahn and Thomas, 1970) and concepts of Roche-equipotential in a manner earlier used by (Saini, et al., 2012) to incorporate the effect of differential rotation on the rotationally distorted stellar models. The inner structure of differentially rotating polytropic models with the polytropic indices 1.5, 2.0, 3.0 and 4.0 have been computed through various physical parameters with suitable combination of the parameters b_1, b_2 and b_3. To determine the equilibrium structures of differentially rotating polytropic models of star a, general problem is investigated.

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As per literature, primarily theory based on distorted polytropes was developed by Chandrasekhar (1933). Since then several authors (Lal et al.,2006; Lal et al., 2012) have addressed themselves to these problems. Kopal (1983), Mohan et al. (1990; 1992; 1994), Saini et al. (2012), Pathania, et al. (2013), have observed that the actual equipotential surfaces of a rotationally and tidally distorted star are approximated by equivalent rotationally and tidally distorted Roche equipotentials. Lal et al. (2005; 2006), have applied this approach on white dwarf stars as well as polytropic stars and hence developed a modelling to determine their equilibrium structures. In this approximation, averaging approach of Kippenhahn and Thomas (1970) and results of the Roche equipotentials obtained by Kopal (1983) are used to incorporate the rotational and tidal effects up to second order of smallness in the stellar structure equations. Mohan et al. (1992) given their contribution for determining the equilibrium structures of differentially rotating or tidally distorted gaseous spheres. Lal et al. (1994) considered the possibility of using this approach to obeying a generalized differential rotation. Once the Roche equipotential surfaces of a differentially rotating star are approximated by modified Roche-equipotential, the approach used by Saini et al. (2012), may now be used to evaluate explicitly the values of modified physical parameter $S_\psi, V_\psi, \overline{g}$ and $\overline{g}^{-1}$.

2. Proposed Law of Differential Rotation

The law of differential rotation used in the present paper is

$$\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4, \qquad (1)$$

where $s = r\sin\theta$ is a non-dimensional measure of the distance of a fluid element from the axis of rotation passing through its centre. This law can also be obtained by the expansion of Taylor series of this form $\omega^2 = f(s^2)$ up to second order of smallness. This law generates a variety of differential rotations, which are commonly expected in stars, but is also in a form, which it can be conveniently subjected to the type of mathematical analysis, which is carried in this paper. Clement (1969) had also represented his study on the oscillation of stars.

2.1 Stability of Differential Rotation

In principle, the instantaneous angular momentum distribution within a star should be calculable from initial conditions. Obviously, this is an impossible task at the present level of knowledge of subject, even when the initial conditions are known. The alternate procedure, which is now widely used, is to choose a study angular momentum distribution by ruling out those of rotating configurations that are (dynamically and thermally) unstable, as well as those that do not comply with the simultaneous conditions of mechanical and thermal equilibrium.

According to Solberg criterion, the configurations rotating with some prescribed angular velocity $\omega = \omega(s,z)$, degenerate into pseudo-barotropic configuration, and the stability condition becomes

$$\frac{d}{ds}\{s^2.\omega(s)\} > 0. \qquad (2)$$

Therefore, in stable homentropic star (i.e. grade $S = 0$, S being the entropy per unit mass) the angular momentum per unit mass must necessarily increase outward. It generalizes to homentropic bodies the well know Rayleigh criterion for an inviscid and incompressible fluid. As was shown by Randers, the stability criterion (2) may be easily explained by the

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conservation of angular momentum per unit mass of each fluid particle, when it is slightly displaced from its equilibrium position. Indeed, when condition (2) is obtained, any mass element moving outward lacks angular momentum compared to the angular momentum of the material in equilibrium about its new location. This lack of angular momentum means a lack of centrifugal force, and a resulting deceleration of the out ward motion. Similarly, if a mass element implied inward, the excess of angular momentum moving inwards tends to derive the fluid particle out again thereby stopping the inward motion.

The results obtained for axis-symmetric motion of Solberg have been later summarized in the form of certain proposition by (Hoiland, 1941). His criterion says that baroclinic star in permanent rotation is dynamically stable with respect to axis-symmetric models if and only if following conditions are satisfied: -

- The entropy per unit mass S never decreases outward.
- On each surface S = constant, the angular momentum per unit mass Ω_s^2 increases as move from the poles to equator.

In a similar manner, Stoeckly (1965) suggested a stability criterion for a differentially rotating model against local perturbation. According to this criterion a model rotating differentially according to the law $\omega = \omega(s)$, is stable if $\frac{d}{ds}\left\{s^2 .\omega(s)\right\} > 0$, for all s from centre to surface.

For a star rotating differentially according to the law (1), to be stable according to Stoeckly criteria (1), must be non-negatives for all values of s inside the star. The stability of each of differential rotations considered by us (given in table 1) has been analyzed according to this criterion and the results of this analysis are presented in the same table.

3. The Roche-Equipotentials of Differentially Rotating Gaseous Sphere Including the Effect of Mass Variation on the Potential

The concept of Roche- equipotential and Roche limit has often been used in literature to investigate the physical parameters and stabilities of binary stars. While computing Roche equipotentials, the whole mass of the sphere is assumed to be concentrated at its centre. This approximation, through reasonably correct for highly centrally condensed stellar models, is not true for the stars which are not highly condensed on the centre. The concept of Roche equipotentials, therefore, needs to be modified in case of stars which are not highly centrally condensed taking into account the effect mass variation on its equipotentials surfaces inside the star. The result on Roche equipotential based on this modification and which are practical interest to the present study are summarized below.

M_0 is assumed to be the entire mass of the differentially rotating primary component and it is more massive than its companion star of mass M_1 (i.e., $M_0 > M_1$). Suppose R is the mutual separation between these two masses from center to center and the position of the two components of this binary system is referred to a rectangular system of Cartesian co-ordinates which have the origin at the center of gravity of mass M_0, the x axis along the line joining the centers of the components, the z axis perpendicular to the plane of the orbit of the two components, $M_0(r)$ is the interior mass of the primary component. The total potential Ω due

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to the gravitational, rotational and other disturbing forces acting an arbitrary point $P(x, y, z)$ may be expressed as:

$$\Omega = G\frac{M_0(r)}{r} + G\frac{M_1}{r_1} + \frac{1}{2}\omega^2\left((x - \frac{M_1 R}{M_0 + M_1})^2 + y^2\right), \tag{3}$$

where $r^2 = x^2 + y^2 + z^2$, and $r_1^2 = (R - x)^2 + y^2 + z^2$.

The distances of point p from the centre of gravity are represented through r and r_1. Total potential Ω is the sum of potentials arising from the mass of the primary component M_0, disturbing potential arising by its companion star of mass M_1 and potential arising from the centrifugal force (Figure 1).

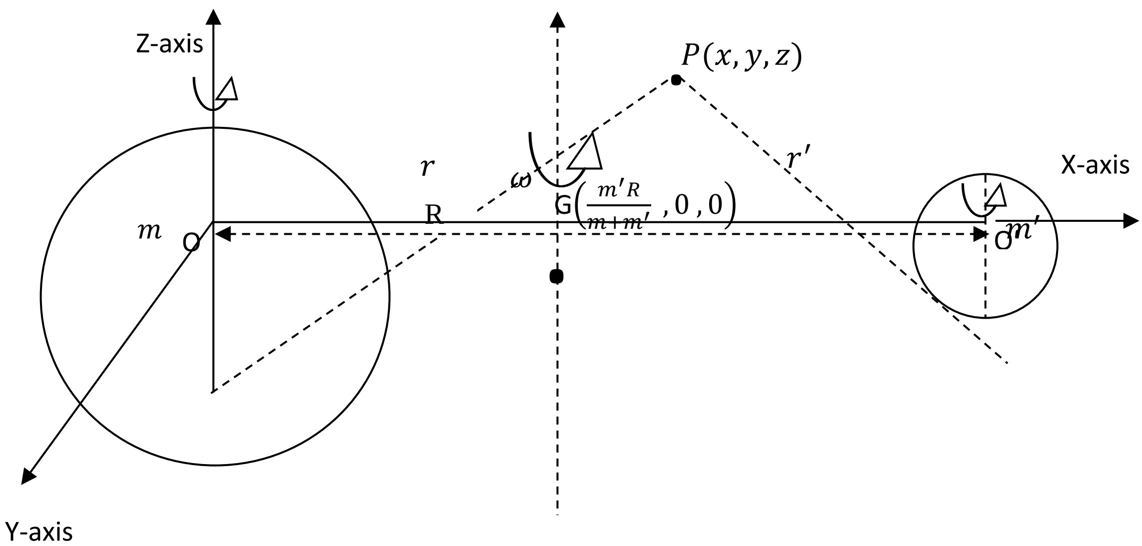

Figure 1. Axis of reference for Roche co-ordinates

The angular velocity ω is identical with the Keplarian angular velocity in close binary system, so that $\omega^2 = G\frac{M_0 + M_1}{R^3}$ If we insert relations (1) in (2) the equation (2) may be expressed in terms of polar spherical coordinates:

$$x = r\cos\phi\sin\theta = r\lambda \quad y = r\sin\phi\sin\theta = r\mu,\ z = r\cos\theta = r\nu \tag{4}$$

$$as, \quad \psi = \frac{1}{r} + q\left[\frac{1}{(1 - 2\lambda r + r^2)^{1/2}} - \lambda r\right] + \frac{1}{2}\omega^2 r^2 (1 - \nu^2), \tag{5}$$

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where $\psi = \frac{R\Omega}{GM_0} - \frac{M_1^2}{2M_0(M_0+M_1)}$, $q = \frac{M_1}{M_0}$ and $t = \frac{M_0(r)}{M_0}$ are non-dimensional parameters and ω^2 is non-dimensional unit of $\frac{GM_0}{R^3}$.

The surfaces generated by setting ψ=constant is represented by left hand side of equation (5) and referred as Roche equipotential. These approximate the equipotential surfaces of a star in a binary system. The form of Roche equipotential depends entirely upon the value of ψ and corresponding equipotential consists two separate ovals, if ψ is large, closed around each of the two mass points. The right hand side of equation (5) can be large only if r or $r_1 = \left(1 - 2\lambda r + r^2\right)^{1/2}$ becomes small it must be nearly equal to r and r_1, if the right hand side of equation (5) is to be constant. Therefore, large values of ψ correspond to equipotentials which differ little but spheres. With reduction in the value of ψ in the expression (5), the ovals define by expression (5) become increasingly elongated in the direction of the centre of gravity of the system until for a certain critical value of ψ (characteristic of each mass ratio), both ovals will unite in a single point on the x-axis to form a dumb-bell like configuration. These limiting value of ψ are called Roche limits. Two ovals of their Roche limit filling by any pair of stars are called contact binaries. The connecting part of dumb-bell open up for still smaller values of ψ and the corresponding equipotential surfaces envelope both the bodies.

4. Physical Parameters $S_\psi, V_\psi, \bar{g}$ and $\bar{g}^{-1}$ of Stellar Models

Expressions for volume, surface area and other physical parameters of differentially rotating polytropic models of a star are investigated. For obtaining the expressions of physical parameters of a polytropic star, we used the approach earlier used by Saini et al. (2012) for obtaining the equilibrium structures of differentially rotating and tidally distorted Prasad models including the effect of mass variation inside the star, and Lal et al. (2006) for obtaining the equilibrium structures of polytropic stars having differential rotation. For computing the distortion effects, the actual equipotentials surface of star are approximated by Roche-equipotentials and (Kopal's, 1983) result on the Roche-equipotentials are then used to express the problem in a form convenient for numerical work. In order to introduce the concept of Roche-equipotentials, assume a mass *M* and radius *R*, for rotating configuration, the total potential Ω of a fluid element is given by the equation of hydrostatic equilibrium may be written in the form

$$d\Omega = dV + \frac{1}{2}\omega^2 d(s)^2 \text{ or } \Omega = V + \int \omega^2(s) s ds,$$

i.e. $$\Omega = \frac{GM_0(r)}{r} + \int \omega^2(s)\, s ds \qquad (6)$$

Assuming Roche model for a differentially rotating gaseous sphere the gravitational potential $V = \frac{GM_0(r)}{r}$, at a point p at distance r from the centre. $M_0(r)$ is mass interior to sphere of

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radius r and M_0 is the total mass of the rotating gas sphere. Substituting these in (6) and multiplying $\frac{R}{GM_0}$, it reduced as:

$$\psi = \frac{t}{r/R} + \frac{1}{2}\frac{R}{GM_0}\omega^2 d(s^2), \tag{7}$$

where $t = \frac{M_0(r)}{M_0}$.

Since dimension of s is same as that of R, assuming ω^2 to have a dimension of $\frac{GM_o}{R^3}$ .the non-dimensional form of (7) can be represented as :

$$\psi = \frac{t}{r} + \frac{1}{2}\int \omega^2 d(s^2) \ . \tag{8}$$

Using, $\omega^2(s) = b_1 + b_2 s^2 + b_3 s^4$ with $s^2 = r^2(1-\nu^2)$ in (8) we have

$$\psi = \frac{t}{r} + \frac{1}{2}b_1^2 r^2(1-\nu^2) + \frac{1}{2}b_1 b_2 r^4(1-\nu^2)^2 + \frac{1}{6}(2b_1 b_3 + b_2^2) r^6 (1-\nu^2)^3 + \frac{1}{4}b_2 b_3 r^8 (1-\nu^2)^4$$
$$+ \frac{1}{10}b_3^2 r^{10}(1-\nu^2)^5 \ . \tag{9}$$

In absence of rotation $b_1 = b_2 = b_3 = 0$, the Roche equipotential (9) reduced to $\psi = \frac{t}{r}$ and if we assume solid body rotation $b_2 = b_3 = 0$ and $t = 1$, equation (9) is reduced to the expression given by (Mohan et al, 1978). Now, ψ is the non-dimensional form of the total potential: $\Omega(\psi) = \frac{R\Omega}{GM}, \lambda = \sin\theta, \sin\phi, u = \cos\theta,$

With the assumption $\psi =$constant (Kopal, 1983) developed the Roche-equipotential and his approach and analysis is used here to develop the relation for co-ordinates (s, ϕ, z) of an element of Roche-equipotential as:

$$r = r_0 R\Big[1 + \frac{1}{2t}b_1 r_0^3 x + \frac{1}{4t}b_2 r_0^5 x^2 + \frac{3}{4t^2}b_1{}^2 r_0^6 x^2 + \frac{1}{6t}b_3 r_0^7 x^3 + \frac{1}{t^2}b_1 b_2 r_0^8 x^3 + \frac{3}{8t^3}b_1^3 r_0^9 x^3$$
$$+ \frac{5}{48t^2}\left(8b_1 b_3 + 3b_2^2\right) r_0^{10} x^4 + \ldots\Big], \tag{10}$$

where $r_0 = \frac{t}{\psi}$ and $x = (1-\nu^2)$. In above equation, terms of t, b_1, b_2, b_3 and r_0 are retained up to third and tenth order of smallness, respectively. The shapes of various equipotential surfaces of differentially rotating gaseous spheres have obtained by setting $r =$constant. In equation (10) R is the radius of undistorted model.

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Following (Kopal , 1983), (Mohan et al., 1990) and (Lal et al.,2006), the explicit expression for S_ψ, V_ψ, $\bar{g}$ and $\overline{g^{-1}}$, using the law of differential rotation given by equation (1) are obtain as

$$V_\psi = \frac{4\pi}{3} r_0^3 R^3 \left[1 + \frac{1}{t} b_1 r_0^3 + \frac{2}{5t} b_2 r_0^5 + \frac{8}{5t^2} b_1^2 r_0^6 + \frac{8}{35t} b_3 r_0^7 + \frac{12}{7t^2} b_1 b_2 r_0^8 + \frac{8}{5t^3} b_1^3 r_0^9 + \frac{16}{105t^2} \left(8b_1 b_3 + 3b_2^2 \right) r_0^{10} + \dots \right]. \tag{11}$$

Following the averaging technique of (Kippenhahn and Thomas,1970) for a topologically equivalent sphere of radius r_ψ, we have the relation:-

$$V_\psi = \frac{4}{3} \pi r_\psi^3 \tag{12}$$

On inserting (11) in (12), we get

$$r_\psi = r_0 R \left[1 + \frac{1}{3t} b_1 r_0^3 + \frac{2}{15t} b_2 r_0^5 + \frac{19}{45t^2} b_1^2 r_0^6 + \frac{8}{105t} b_3 r_0^7 + \frac{152}{315t^2} b_1 b_2 r_0^8 + \frac{97}{405t^3} b_1^3 r_0^9 + \left(\frac{16}{45t^2} b_1 b_3 + \frac{212}{1575t^2} b_2^2 \right) r_0^{10} + \dots \right] \tag{13}$$

The surface area of Roche equipotentials surface $\psi =$ constant is given by

$$S_\psi = 4\pi r_0^2 R^2 \left[1 + \frac{2}{3t} b_1 r_0^3 + \frac{4}{15t} b_2 r_0^5 + \frac{14}{15t^2} b_1^2 r_0^6 + \frac{16}{105t} b_3 r_0^7 + \frac{36}{35t^2} b_1 b_2 r_0^8 + \frac{34}{35t^3} b_1^3 r_0^9 + \frac{88}{945t^2} \left(8b_1 b_3 + 3b_2^2 \right) r_0^{10} + \dots \right] \tag{14}$$

The explicit expressions of gravity $\bar{g}$ and its inverse $\overline{g^{-1}}$ can be shown to be respectively.

$$\bar{g} = \frac{tGM_\psi}{r_0^2 R^2} \left[1 - \frac{4}{3t} b_1 r_0^3 - \frac{4}{5t} b_2 r_0^5 - \frac{7}{9t^2} b_1^2 r_0^6 - \frac{64}{105t} b_3 r_0^7 - \frac{488}{315t^2} b_1 b_2 r_0^8 - \frac{134}{945t^3} b_1^3 r_0^9 - \frac{16}{4725t^2} \left(505 b_1 b_3 + 162 b_2^2 \right) r_0^{10} + \dots \right] \tag{15}$$

$$\overline{g^{-1}} = \frac{r_0^2 R^2}{tGM_\psi} \left[1 + \frac{4}{3t} b_1 r_0^3 + \frac{4}{5t} b_2 r_0^5 + \frac{131}{45t^2} b_1^2 r_0^6 + \frac{64}{105t} b_3 r_0^7 + \frac{1352}{315t^2} b_1 b_2 r_0^8 + \frac{1198}{189t^3} b_1^3 r_0^9 + \frac{16}{4725t^2} \left(1145 b_1 b_3 + 432 b_2^2 \right) r_0^{10} + \dots \right] \tag{16}$$

In the above expressions, M_ψ is the mass contained with Roche- equipotential.

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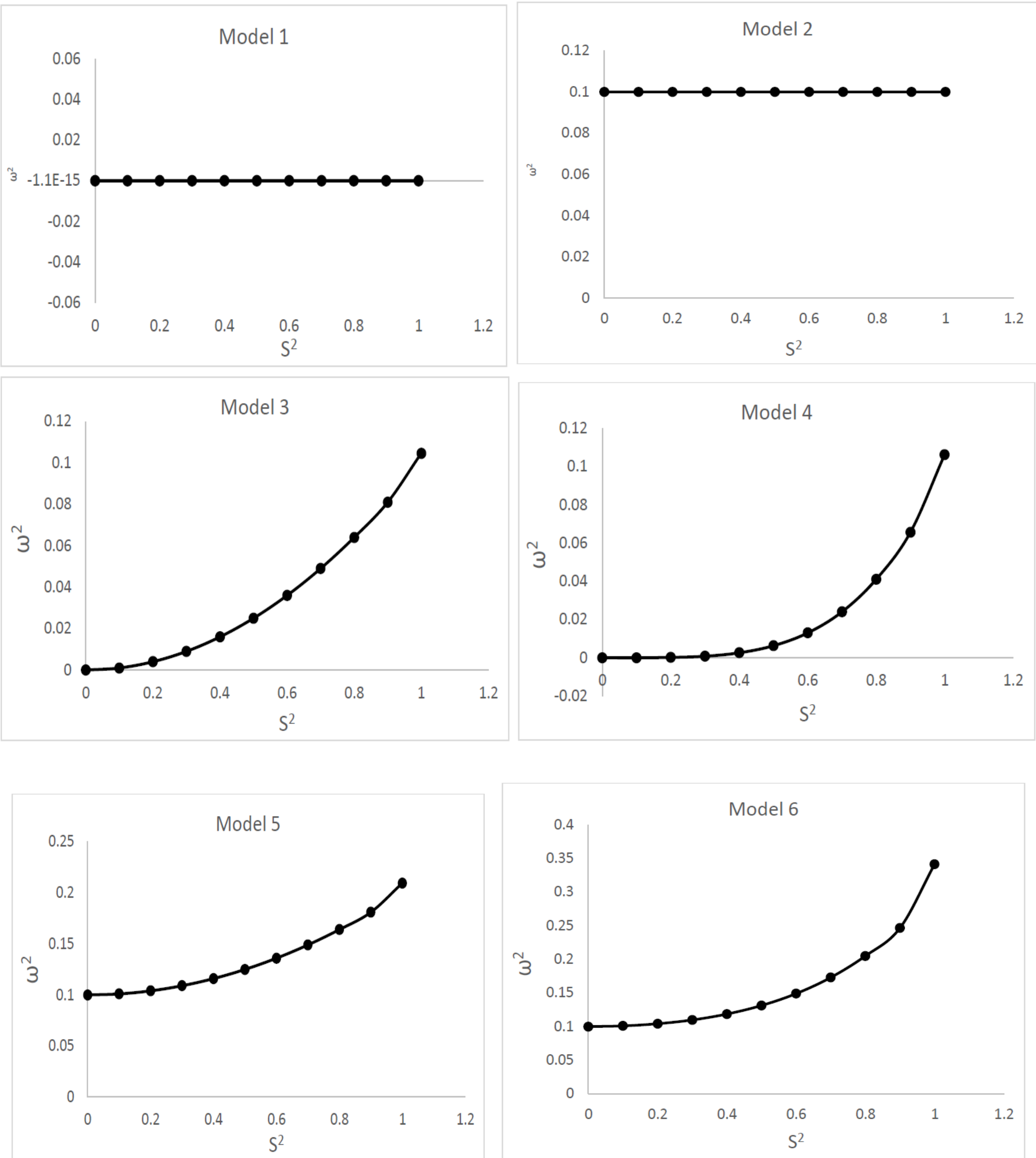
Table 1. Behaviour of angular velocity for certain differentially rotating stellar models

Model Number	Values of various parameters in the law of differential rotation $\omega(s)=b_1+b_2s^2+b_3s^4$			Behavior of the square of the angular velocity ω^2 from axis of rotation (s=0) to equator(s=1) in the equatorial plane/(from pole to equator on the surface), for a differentially rotating model in which s is in the equatorial radius R_e	Stability of the model according to Stoeckly criterion
	b_1	b_2	b_3		
1	0.0	0.0	0.0	Non-rotating model	Stable
2	0.1	0.0	0.0	Solid body rotation about axis of rotation in which ω^2 is 0.1 throughout the model	Stable
3	0.0	0.1	0.0	ω^2 increases gradually from 0 to 0.104651	Stable
4	0.0	0.0	0.1	ω^2 increases first slowly and then more rapidly from 0 to 0.106294	Stable
5	0.1	0.1	0.0	ω^2 increases gradually from 0.1 to 0.209551	Stable
6	0.1	0.1	0.1	ω^2 increases rapidly from 0.1 to 0.341509	Stable
7	0.1	0.2	0.1	ω^2 increases still more rapidly from 0.1 to 0.476902	Stable
8	0.1	-0.05	0.0	ω^2decreases gradually from 0.1 to 0.048772	Stable
9	0.1	-0.1	0.05	ω^2 decreases first slowly than rapidly and then again slowly from 0.1 to 0.050027	Stable
10	0.1	-0.15	0.1	ω^2 decreases first slowly from the value 0.10at s=0 but later starts increasing to the value 0.051297 at s=1	Stable
11	0.1	-0.02	0.4	ω^2 first remains practically constant at about 0.1 value from s=0 to s=0.3 and then starts rapidly increasing to the value 0.645424 at s=1	Stable
12	0.04	-0.01	0.0625	ω^2 first remains practically constant at about 0.04 value from s=0 to s=0.45 and then starts rapidly increasing outward to the value 0.096754 at s=1	Stable
13	0.1	0.02	-0.05	ω^2 increases first slowly from its value 0.1 at s=0 up to s=0.45 and then decreases to the value 0.066944 at s=1	Stable
14	0.1	-0.1	0.0	ω^2decreases from its value 0.1 at s=0 to value 0 for s=1	Unstable
15	0.04	-0.16	0.16	ω^2 first decreases from its value 0.04 for s=0 to the value 0 at s=0.7 and then again increases rapidly outwards to the value 0.042812 at s=1	Unstable


Model 1
Model 2
Model 3
Model 4
Model 5
Model 6
ω²
S²
-1.1E-15

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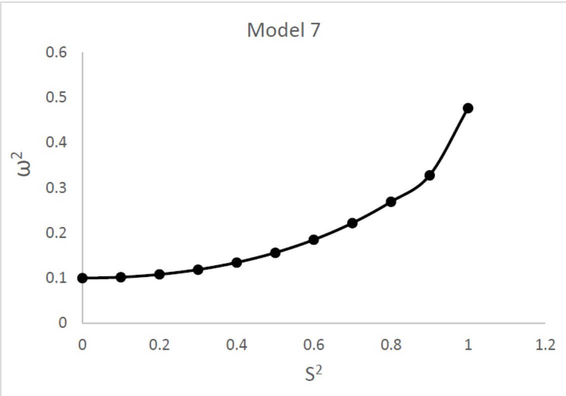

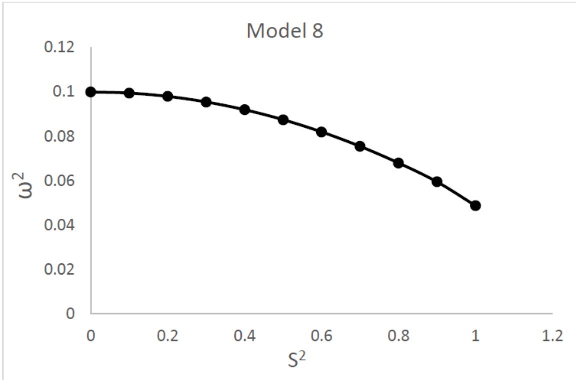

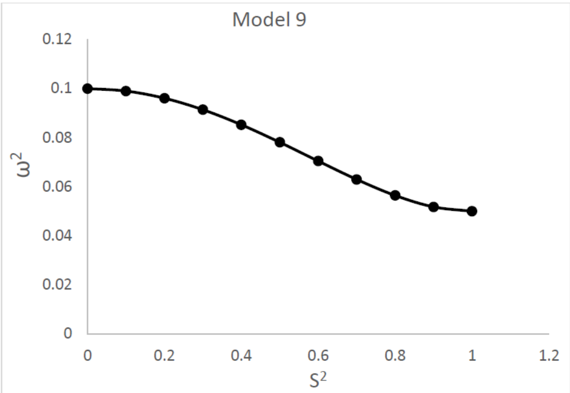

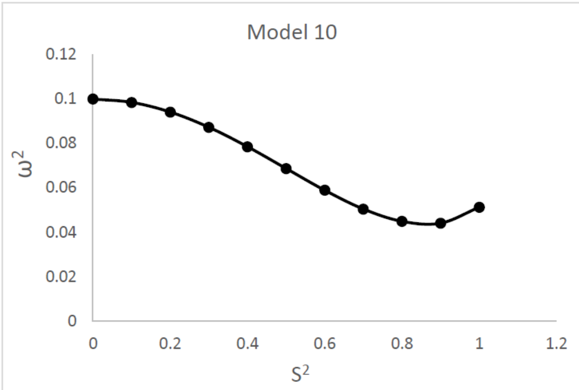

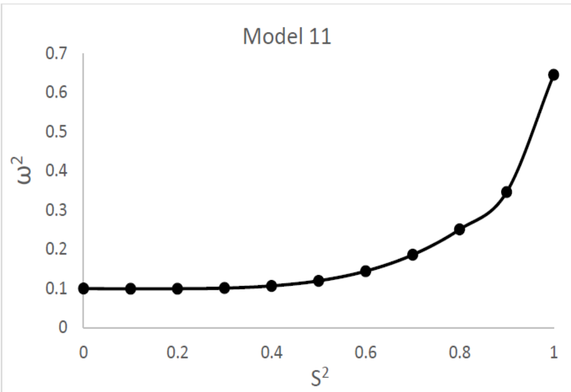

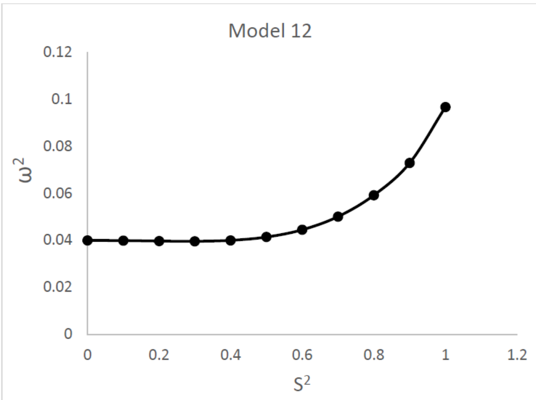

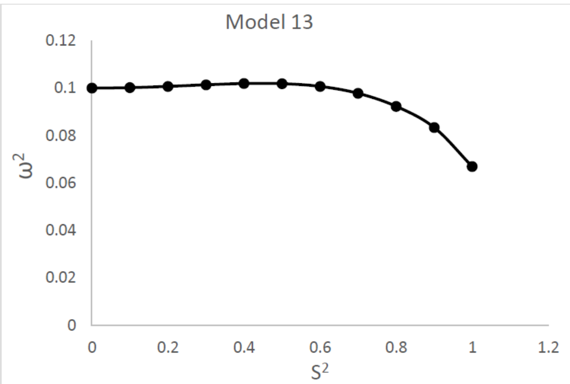

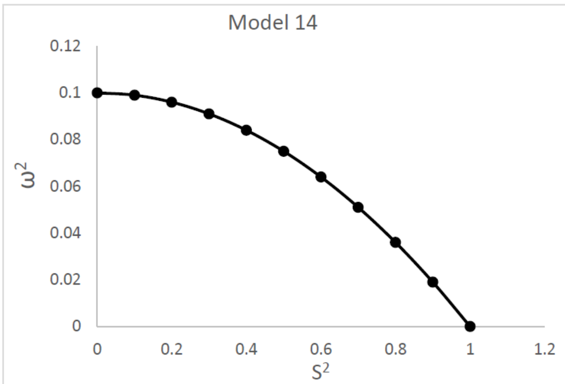

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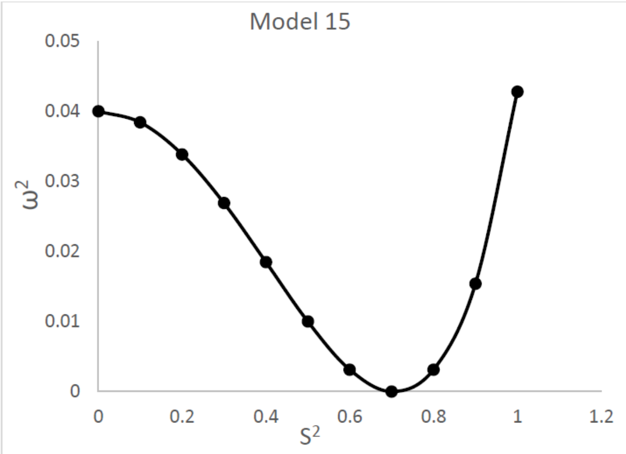

Figure 2. Graphs of ω^2 versus s^2 certain differentially rotating stellar models

5. Conclusion

The present work assumes the law (1) of differential rotation in which 's' is assumed to be the non-dimensional measure of distance and b_1, b_2 and b_3 are constants. Stability of various type of polytropes have also been analyzed in this paper by using Stoeckly criterion, presented by equation (2). Stability and behavior of angular velocity of some models have been represented in Table 1. We also used Stoeckly criterion to examine weather assumed stellar models in Figure 2 are dynamically stable or not. Using several idealized models for the non-uniform density within primordial gas clouds we are now able to compute the appropriate polytropic structures of different indices.

Study of stability is considered to be dependent upon the variation of angular velocity of particle with variation in *'s'*, which is the distance of particle from center. Variations in angular velocities of these models along with distance have also been represented through the graphs. Roche equipotential derived in this paper for obtaining the structure and stability of differentially rotating gaseous spheres provides valuable information about the variation of various structural identities with radius. Roche equipotential assumes the entire mass of star to be concentrated at the center of sphere which provides a macro structure of star by assuming very less influence of convection and turbulences. Due to this assumption present study becomes less useful in case of highly turbulent and active star. After obtaining the structure, explicit expressions of modified distortion parameters have also been obtained by using the approach of averaging approach of (Kippenhahn and Thomas, 1970) and results of the Roche equipotentials obtained by (Kopal, 1983) are used to incorporate the rotational and tidal effects up to second order of smallness in the stellar structure equations.

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