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Theoretical and Experimental Estimation of Uncertainty of Measurement during Calibration of CMM

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Abstract
In the present paper we briefly discuss the universal (theoretical) and experimental methods of estimation of uncertainty of measurement. As an application we have taken the case of manual 3D co-ordinate measuring machine of with 1 μm least count and this machine has been calibrated with the help of a Step Gauge. The study concludes that both universal and experimental methods give fairly identical results and hence universal method can be used easily for the uncertainty calculation of such machines. This is first of its kind of work where theoretically estimated uncertainty value has been compared w.r.t. the practically observed value.

Keywords- CMM, Calibration, Uncertainty, Length Measurement, Probability Distribution.

1. Introduction
To establish the suitability of a 3D Co-ordinate Measuring Machine (CMM) for length measurement it is necessary to estimate and specify the length measurement uncertainty of the machine. Uncertainty estimation is an essential part of CMM calibration. Measured value may be slightly less or more than the true value of the physical quantity and the range in which the true value of measurand is estimated to lie is called the uncertainty of measurement. Quality of end product largely depends upon the quality of measurement; hence measurement is considered to be the key process especially in the field of production. Any measurement process accompanies certain uncertainty with it. There can be many assignable and unassignable causes which influence the measurement process and one of the most significant cause associated with the measurement is the `uncertainty of measurement'. In the present competitive era of specialised technologies quality has become an essential requirement for survival which can only be ensured through proper measurement techniques with known uncertainty. Calibration of 3D CMM has a special significance since these machines are used for the inspection of components which serve as future standards.

1.1 Calibration of CMM
Precise physical bodies of known length can be used as reference standards for checking length measurement uncertainty of instruments having mechanical sensors. This fundamental task in metrology is of prime importance since in practice the majority of measuring

requirements are for the measurement of length. This method has been utilised for calibrating 3D CMM of SIP Switzerland make using the step gauge of KOBA, Germany make. The step gauge is of castellated configuration and in it a large number of forward and backward facing gauge faces are lined up along a single line of measurement. This line of measurement is same for measurements between any two faces. The gauge faces are well protected and the actual gauge points are situated on the neutral fibre of the holder and this means that there are no first-order changes in length if the state of bending changes.

1.2 Length Measurement Uncertainty of CMM

Length measurement uncertainty is defined as the uncertainty with which a CMM can determine the distance between two points located on parallel surfaces. The length measurement uncertainty u is specified in a simplified form as a length-dependent parameter

$$u = A + KL \leq B \tag{1}$$

here A, K and B are constants and L is the measured length (Figure 1) (Catalogue No. 6100/E/01/2000, 2000).

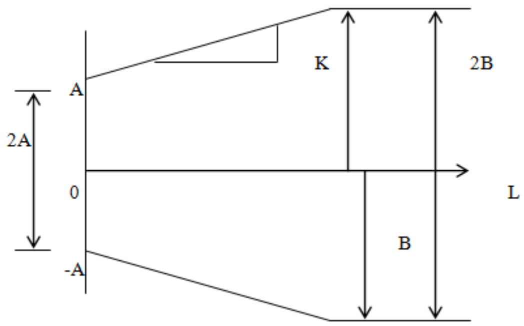

Figure 1. Expression of uncertainty VS length measured

In our present discussion, we have taken into account only one-dimensional length measurement uncertainty (1D length measurement uncertainty), for which the equation (1) can be written in the following form.

$$u_1 = A_1 + K_1 L_1 \leq B_1 \tag{2}$$

This is valid for measurement of the distance between parallel surfaces whose surface normal lines are, with a good approximation, in a coordinate line of the measuring machine. One-dimensional length measurement uncertainty can be evaluated by two means. Firstly, by theoretically calculating the overall uncertainty taking into account all the main contributory factors which can influence the correctness of measurement results, secondly, by actually

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plotting the deviation of measured data from true value and thus graphically ascertaining the measurement uncertainty. As shown by the above relationship, length measurement uncertainty increases with the length measured, hence for our discussion we have taken two cases, Case I for IInd minimum length i.e. 39.932 mm and Case II for IInd maximum length i.e. 359.724 mm and results obtained by theoretical and graphical methods have been compared for these lengths.

2. Uncertainty Estimation by Universal Method

In the present day era of global trade, it is extremely essential to adopt an internationally agreed common method for evaluation and expression of uncertainty of measurement. In view of the above requirement, at the initiative of International Committee for Weights and Measures (CIPM), a universal method was suggested by International Bureau of Weights and Measures (BIPM) 'Working Group' on the 'Statement of Uncertainty'. As we all know that the Conventional Method of uncertainty evaluation is being replaced by this new 'Universal Method', we have used 'Universal Method', for our calculation of measurement uncertainty. The uncertainty of the result of a measurement generally consists of several components, which may be grouped into two categories according to the method used to estimate their numerical values (Taylor et al., 1994).

Type A: those which are evaluated by statistical method (random uncertainties).
Type B: those which are evaluated by other means (systematic uncertainties).

2.1 Type 'A' Evaluation of Standard Uncertainty

Type `A' evaluation of standard uncertainty is based on statistical method and is calculated from the standard deviation of the mean of a series of independent observations. To evaluate one-dimensional length measurement uncertainty of CMM, step gauge is aligned along x axis and 19 successive faces were measured with five observations of each length. Measurements are taken up to 380 mm as maximum range of CMM along x axis is 400 mm. Measurement data is given in the Table 1.

Type 'A' inputs are usually linked to repeatability. In our experiment we have taken 5 observations for each length under same conditions of measurement. For each length Type-A standard uncertainty U_A associated with the mean value ($\bar{q}$) can be calculated from the estimated standard deviation of the mean (NABL, 1994; Taylor et al., 1994) i.e.

$$U_A = s(\bar{q}) = = \left[\frac{1}{n(n-1)}\sum_{i=1}^{n}(q_i - \bar{q}_i)^2\right]^{\frac{1}{2}} \quad (3)$$

Here,

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$n = number\ of\ observations$

$q_i = individual value\ of\ length measured$

$\bar{q}_i = mean\ value$

Mean value $\bar{q}$ is given by

$$\bar{q} = \frac{1}{n}\sum_{i=1}^{n} q_i \qquad (4)$$

From equation (3) Standard Uncertainty for nominal length 39.932 mm is obtained as $U_A = s(\bar{q}) = 0.245$ µm.

Table 1. Measurement data collected

S. No	Nominal length (in mm)	Observations (in mm)				
		I	II	III	IV	V
1.	20.000	20.000	20.000	20.000	20.001	20.000
2.	39.932	39.933	39.933	39.932	39.932	39.933
3.	59.933	59.933	59.932	59.933	59.933	59.933
4.	79.927	79.928	79.928	79.928	79.928	79.928
5.	99.928	99.928	99.929	99.928	99.929	99.929
6.	119.913	119.915	119.913	119.913	119.914	119.915
7.	139.914	139.914	139.914	139.913	139.916	139.915
8.	159.919	159.920	159.920	159.920	159.920	159.920
9.	179.919	179.919	179.919	179.919	179.920	179.920
10.	199.872	199.873	199.873	199.873	199.873	199.873
11.	219.873	219.873	219.874	219.875	219.874	219.875
12.	239.815	239.816	239.816	239.816	239.816	239.816
13.	259.815	259.816	259.816	259.815	259.817	259.816
14.	279.741	279.743	279.742	279.742	279.742	279.742
15.	299.741	299.742	299.741	299.741	299.744	299.743
16.	319.729	319.729	319.728	319.729	319.729	319.729
17.	339.729	339.731	339.729	339.728	339.732	339.729
18.	359.724	359.726	359.725	359.724	359.725	359.725
19.	379.725	379.726	379.724	379.724	379.728	379.727

2.2 Type 'B' Evaluation of Standard Uncertainty

This type of evaluation of standard uncertainty is based on scientific judgement using all the relevant information available, which may include (Taylor et al., 1994) previous measurement data, experience with or general knowledge of the behaviour and property of relevant materials and instruments, manufacturer's specifications, data provided in calibration and other reports, uncertainties assigned to reference data taken from handbooks.

In our experiment, following are the possible factors identified which contribute significantly in the uncertainty of measurement.

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1. Uncertainty due to deviation of mean calibration temperature from 20^0 C on account of different co-efficient of linear expansion ($\propto$) of step gauge and CMM.

Here,
$\propto$ (step gauge) $=11.5 \times 10^{-6}$ mm/^{0}C,
$\propto$ (CMM) = 10.5×10^{-6} mm/ ^{0}C,
Starting calibration temp^r = 17^0 C,
End calibration temp^r = 18^0 C,
Mean calibration temp^r = 17.5^0 C.

Uncertainty contribution (a_1)
a_1 = (length measured) x (deviation of mean calib. temp^r from 20^0 C) x (difference in $\propto$ values)
$= 0.0399 \times (20 - 17.5) \times (11.5 \times 10^{-6} - 10.5 \times 10^{-6})$
= 0.0998 μm.

This uncertainty contribution (a_1) follows a rectangular probability distribution (Shankar, 1999) hence standard uncertainty due to a1 is given by
$U_1 = a_1 / \sqrt{3} = 0.0576$ μm.

2. Uncertainty quoted in step gauge calibration certificate
U = (0.2 + 0.8 x L) μm at 95% Confidence Level,

where L is the length measured in meter. Corresponding uncertainty contribution at L = 0.0399 meter
$a_2 = 0.2 + 0.8 \times 0.0399$
= 0.2319μm.

This uncertainty contribution follows a normal probability distribution hence standard uncertainty due to a_2 is given by
$U_2 = a_2 / 2$

(here dividing factor 2 has been taken since confidence level mentioned in standard's calibration certificate is 95%). Hence,
$U_2 = 0.2319/ 2 = 0.116$ μm.

3. Uncertainty contribution due to variation of temperature during calibration (± 0.5 C) for the entire range
a_3 = range X variation in temp^r X $\propto$ value of step gauge
$= 0.0399 \times 0.5 \times 11.5 \times 10^{-6} = 0.229$ μm.

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This follows a rectangular probability distribution, hence standard uncertainty due to a_3 is given by

$U_3 = a_3 / \sqrt{3} = 0.132$ µm.

4. Uncertainty contribution due to resolution of CMM

$a_4 = ½$ (resolution) = 0.5 µm.

This also follows a rectangular probability distribution, hence standard uncertainty due to a_4 is given by

$U_4 = a_4 / \sqrt{3} = 0.289$ µm.

5. As per the uncertainty quoted in the calibration certificate of the step gauge, described in point no. 2 above, if the error in nominal length of step gauge is compared with the slip gauges of equivalent length, the grade of step gauge can be safely assumed to be grade 0, for the purpose of ascertaining flatness and parallelism error (Indian Standard, IS: 2984, 2003; Catalogue No. 1000/97E, 1997).

Uncertainty contribution due to flatness error of measuring faces of step gauge (Indian, 2984)

$a_5 = 0.1$ µm.

As this follows a rectangular probability distribution, standard uncertainty due to a_5 is given by

$U_5 = a_5 / \sqrt{3} =_= 0.0577$ µm.

6. Uncertainty contribution due to parallelism error of measuring faces of step gauge (Indian, 2984)

$a_6 = 0.1$ µm.

With rectangular probability distribution, the standard uncertainty contribution due to a_6 becomes

$U_6 = a_6 / \sqrt{3} = 0.0577$ µm.

2.3 Combined Standard Uncertainty

The combined standard uncertainty of a measurement result Uc, is obtained by combining the individual standard uncertainties, whether arising from a Type `A' evaluation or a Type `B' evaluation, using the usual method for combining standard deviations.

Thus

$Uc = \sqrt{[\, UA^2 + U_1^2 + U_2^2 + U_3^2 + U_4^2 + U_5^2 + U_6^2 \,]}$

$Uc = 0.429$ µm.

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2.4 Expanded Uncertainty

Often it is required to have a measure of uncertainty that defines an interval about the measurement result y within which the value of the measurand Y can be confidently asserted to lie. The measure of uncertainty intended to meet this requirement is termed expanded uncertainty `U' and is obtained by multiplying Uc by a coverage factor `k'.

Thus $U = k.U_c$

And it can be confidently asserted that
$y - U \leq Y \leq y + U$,

which is commonly written as
$Y = y \pm U$.

In general, the value of coverage factor k is chosen on the basis of the desired level of confidence to be associated with the interval defined by U= k.Uc. When the normal distribution applies k = 2 defines an interval having a level of confidence of approximately 95 percent (Taylor et al., 1994; Shankar, 1999). Hence for Case I expanded uncertainty becomes

$U_I = 2 \times 0.429 = 0.858\ \mu m$ (at 95% CL).

Similar calculations can be repeated for Case II i.e. for length 359.724 mm. Expanded uncertainty for Case II becomes
$U_{II} = 2.796\ \mu m$ (at 95% CL).

2.5 Uncertainty Estimation by Experimental Method

In magnitude, length measurement uncertainty is given as the difference between the length value La indicated by the co-ordinate measuring machine or printed or displayed by its output processor and the true value Lr given by calibration certificate of step gauge, i.e.

$|La - Lr| \leq u$ in at least 95% of all cases Here La can be both larger and smaller than Lr For the purpose of graphic analysis, the differences $\Delta L = La - Lr$ are found and these differences are plotted, with the correct signs, for the individual measured lengths and runs in a length measurement uncertainty grid (Figure 2). The top and bottom boundary lines produce a funnel shaped outline with the neck of the funnel measuring 2A (where A is a figure specified for length measurement uncertainty irrespective of length). 95% of all the test measurements must lie within or on the boundaries. A quantitative analysis is made simply by counting the number of measurements which lie outside the boundary lines. Right side of Figure 2 shows the frequency plot which is a normal curve. Mean value of deviation comes 0.726μm, which indicates the error. This error can be compensated from all the measured values.

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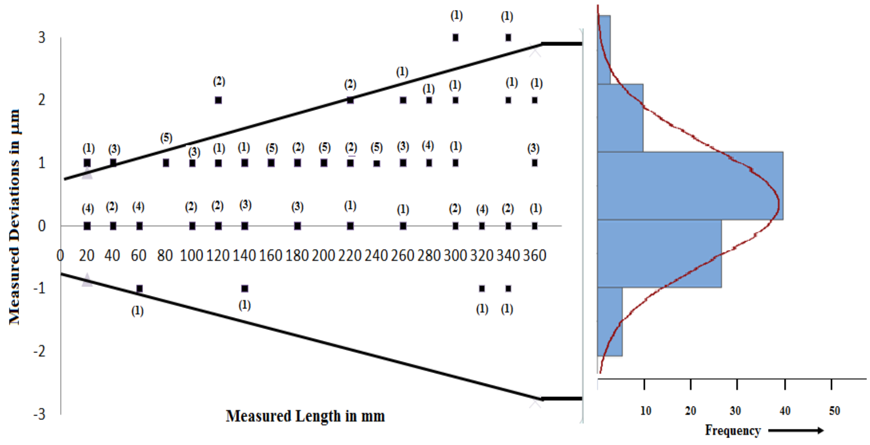

Figure 2. Measurement error plot and associated normal curve

3. Conclusion

If top and bottom boundary lines in Figure 2 are drawn through the two points given by universal method of uncertainty evaluation at 39.932 mm and 359.724 mm taking mean deviation line as datum, it can be seen from the plot that 4 points are falling outside the funnel shaped outline. This satisfies our condition of 95% confidence level since our total observations are 95 and hence 4.75 points may lie outside the funnel shaped outlines. From Figure 2 as well as from the results of universal method, a generalised relationship between `length measured' and `measurement uncertainty' can be derived as per equation (2).

$U_1 = (0.6 + 6.1\ L)\ \mu m \leq 2.8\ \mu m$, where L is in meters.

Experimental method of uncertainty evaluation gives the true uncertainty of measurement process, but this process is quite elaborate and needs several readings to be taken for each step of measurement. The above comparison between `Experimental Method' and `Universal Method' shows that similar results can also be obtained from `Universal Method' with lesser efforts but with careful selection and determination of various contributory factors and hence Universal Method can be used for the evaluation of measurement uncertainty of mechanical measuring instruments.

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