

Dynamic Analysis of a Quarter Car Model Moving Over a Gaussian Bump

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Abstract

Suspension systems have always been an important part of consideration in automobile considering the ride comfort and passenger safety. A modern vehicle utilizes advanced suspension systems and therefore various studies regarding quarter car suspension analysis have been done. The present work on quarter car model passing over a Gaussian bump is carried out to investigate the effect of spring stiffness on the ride comfort considering a speed range of 10 km/hr – 20 km/hr. The equations of the quarter car system are solved by MATLAB and the behavior of the system with different spring stiffness is analyzed with respect to its effect on sprung mass displacement and Root Mean Square (RMS) value of the sprung mass displacement.

Keywords- Dynamics, Quarter Car Model, Modeling and Simulation.

1. Introduction

Suspension system plays a very important role in the ride safety and comfort of the passengers and always been a very competitive factor in the automotive industries. A good suspension system reduces vibrations and shocks coming from the road and provides comfort to the rider and passengers. Vehicle suspension systems are of three type's passive, semi active and active. Generally, passive system is popular owing to its reliability and low cost and comprises of wheels with tires, spring and damper elements. Ride comfort and road holding are the most important factors to be considered for the selection of the suspension parameters like spring stiffness and damping constant etc. Hence, for the better ride comfort of the passenger there should not be too large oscillations/displacements when the vehicle comes in contact with the speed bumps or road irregularities.

Analysis of the suspension system performance can be made using simplified dynamical models of vehicles such as quarter car model or half car model. In such models, the tire is modeled as a spring with or without damping and is always in contact with the road surface. The rotational motion of the tire is neglected. The suspension system is represented by a spring and damper system. The mass of the vehicle supported by the suspension spring is lumped into sprung mass while, the mass of tire, wheel hub and suspension linkages are collectively lumped into unsprung mass.

Hassaan (2014) analyzed a quarter-car model with passive elements. He evaluated the dynamics in terms of acceleration and sprung mass displacement and also determined the effect of car speed and suspension damping on acceleration and sprung mass displacement. In his paper, he showed that the speed of car should not be more than 6.75 km/h for effective ride comfort while passing through a circular hump for the given suspension and damping parameters. The study evaluated and shown the effect of car suspension parameters on the passenger ride comfort when passing over a circular hump.

Hassaan (2015) classified different types of miniature bumps and compared their effect on ride comfort by considering a fixed speed. He also indicated different speed limits depending on different shapes of miniature bumps for maintaining the ride comfort to the passenger. In addition, the author analyzed the performance of the suspension on a novel harmonic hump.

Phalke and Mitre (2017) analyzed the dynamic response of ride performance and road holding using a two degree of freedom quarter-car model. They analyzed the system behavior and examined the effect of damping coefficient, stiffness, sprung mass and velocity on ride comfort and road holding considering a bump height of 0.1m. They observed that with increasing damping RMS acceleration decreases which further decreases the ride comfort. They also showed that with the increasing stiffness value the ride comfort goes on increasing initially then decreases at a particular value and again goes on increasing.

Satyanarayana et al. (2018) presented the control of a quarter car model with optimized passive suspension elements. They used stochastic optimal preview controller for determining the parameters of a passive suspension system like spring stiffness and damping coefficient optimally by the mean square equivalence of control force of the passive suspension. The results showed that the optimized passive control is closely related to active control suspensions and can be used in place of active suspensions having equivalent varying suspension parameters.

Marzbanrad et al. (2013) evaluated the spring and damper settings that ensuring the ride comfort of a vehicle at different speeds and varied the suspension settings for roads having different levels of roughness. Design of experiment method has been utilized for improving the ride comfort.

Cho et al. (2017) analyzed a passive suspension system of a quarter car model to improve handling and ride performance of a vehicle. They examined the two degree and three degree of freedom model with and without tire damping. They used different damping ratios and compared between passive suspension system with tire damping and without tire damping. For effective ride comfort, the results showed that the suspension system of both types with tire damping is more comfortable than suspension system without tire damping.

Jamali et al. (2017) studied the dynamic behavior performance of passive suspension system based on the two degrees of freedom system quarter car model and analyses the frequency responses of the system with different values of the damping constant. He plotted the frequency responses of the system in terms of the ratios of the displacements of sprung and unsprung mass to the displacement of input representing the road disturbance. The ratios of the accelerations for sprung mass and unsprung mass to the road input were also plotted against the non-dimensional frequency which can be used to design a more realistic vehicle suspension system by considering the effect of road disturbances.

Dharankar et al. (2018) presented the analysis of position dependent damping (PDD) and a methodology for estimation of its parameters for passive suspension of vehicles and performed the time domain simulation of a quarter car model with nonlinear PDD for different forward speeds of vehicle and for generated road profile. Road excitation and vehicle vibration characteristics are considered for a detailed analysis of PDD curve.

So far, several models have been developed for a passive system of the quarter car and the speed ranges have been determined for the better ride comfort and passenger safety. The effect of suspension damping on sprung mass displacement and acceleration has been analyzed considering bumps on the road. In the existing methods, quarter car models with two or more degrees of freedom have generally been analyzed; however, a two degrees of freedom model is sufficient to analyze the dynamics of sprung and unsprung masses. Moreover, active suspension systems have been analyzed with feedback systems; but, generally vehicles are built with the passive suspension due to simple and reliable design. In the existing literature, most of the works focused on the effect of suspension damping to improve the performance of the suspension system. However, it is interesting to analyze the effect of spring stiffness on the ride comfort with varying speeds. Therefore, in this paper, the quarter car model is analyzed for determining the ride comfort and evaluating its sensitivity to suspension spring stiffness for different speeds. The ride comfort is measured by the RMS value of the vertical chassis acceleration and its maximum displacement.

2. Methodology

The effect of varying spring stiffness on the performance of suspension is studied using quarter car model as shown in Figure 1. The sprung mass (labeled as M) is supported by the suspension spring and damper labeled in the Figure as k_s and c_s respectively. The mass of wheel, wheel hub and adjoining suspension parts is lumped together in unsprung mass labeled in the Figure as m . This unsprung mass rests on the tire whose spring stiffness and damping are defined as k_t and c_t respectively. The system shows two degrees of freedom i.e., x_1 and x_2 associated with motion of the sprung and unsprung mass, respectively.

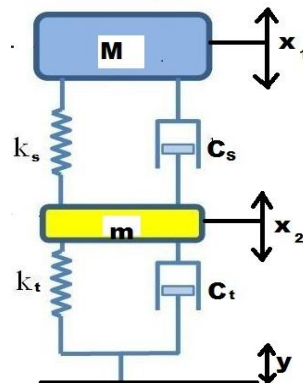


Figure 1. Schematic of the quarter car model

The considered numerical values for various parameters for the quarter car model are given in Table 1.

Table 1. Parameters values

Parameter	Values
M	500kg
m	30kg
ks	20000 N/m
kt	200000 N/m
cs	1500 N-s/m
ct	150 N-s/m

The performance of suspension is examined for the case of a quarter car model driven over a bump. The bump is assumed to be Gaussian and is described by equation (1).

$$y = ae^{((-x-b)^2)/(2c^2)} \quad (1)$$

where, a is the maximum height of the bump, b is the distance of the maximum height of the bump from the starting position and the parameter c controls the width of the bump, see Figure 2. In the present study, a is taken to be 0.05m, b and c are taken to be 10m and 1.2m respectively.

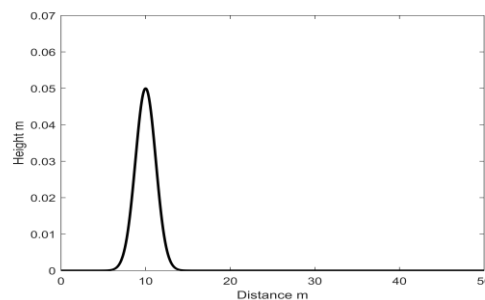


Figure 2. Gaussian bump

The set of differential equations governing the dynamics of quarter car model is given below:

$$\begin{aligned} \dot{v}_2 &= -\frac{k_s}{M}(x_2 - x_1) - \frac{c_s}{M}(v_2 - v_1) - g \\ \dot{x}_2 &= v_2 \\ \dot{v}_1 &= -\frac{k_t}{m}(x_2 - y) + \frac{k_s}{m}(x_2 - x_1) - \frac{c_t}{m}(v_1 - \dot{y}) + \frac{c_s}{m}(v_2 - v_1) - g \\ \dot{x}_1 &= v_1 \end{aligned} \quad (2)$$

where, x_1 , x_2 , v_1 and v_2 represent the position as velocities of the unsprung and sprung mass respectively. To obtain the positions and velocities of the sprung and unsprung mass as a function of time, this set of ordinary differential equations is integrated using Matlab's ODE 45. ODE 45 is an ordinary differential equation solver based on an explicit Runge-Kutta (4, 5) formula of Dormand and Prince (1980).

3. Results and Discussions

The results obtained by integrating the system of ODEs given in equation (2) for the quarter car model moving over a bump at 15km/h are given in Figure 3. Figure 3 (a) shows the sprung mass displacement as a function of time. It can be seen that the sprung mass rises sharply near 2.5 sec as the quarter car model hits the bump. The quarter car model is under-damped, therefore after crossing the bump, sprung mass oscillates for a while and the oscillations eventually die off as the energy is dissipated by the dampers. Similar behavior is displayed by spring compression in Figure 3 (b) and by sprung mass velocity and acceleration in Figure 3 (c) and (d) respectively.

The evaluation of ride comfort in the present study is based on the maximum displacement of sprung mass and RMS value of the sprung mass acceleration. The maximum displacement of sprung mass as considered in this study is given in Figure 4. It is the difference between the maximum and minimum value of the sprung mass displacement. For a good ride comfort, it is desired that both of these quantities should be as low as possible.

The effect of varying suspension spring stiffness (k_s) on the maximum displacement of sprung mass and RMS value of its acceleration is further explored. Figure 5 shows the variation of maximum displacement of sprung mass and RMS value of its acceleration with k_s at a driving speed of 15 km/h. It can be seen that both of these quantities are sensitive to change in suspension spring stiffness. The maximum sprung mass displacement shows higher values at lower spring stiffness and falls sharply as the spring stiffness is increased. This is likely due to the fact that a softer spring could compress more and hence result in greater sprung mass displacement. On the other hand, RMS acceleration is lower for softer spring and rises as the spring stiffness increases. The rise saturates at a spring stiffness of around

30,000 N/m and falls slowly afterwards. Considering such variation of these quantities with k_s it seems that selecting a higher value of k_s will be an appropriate choice.

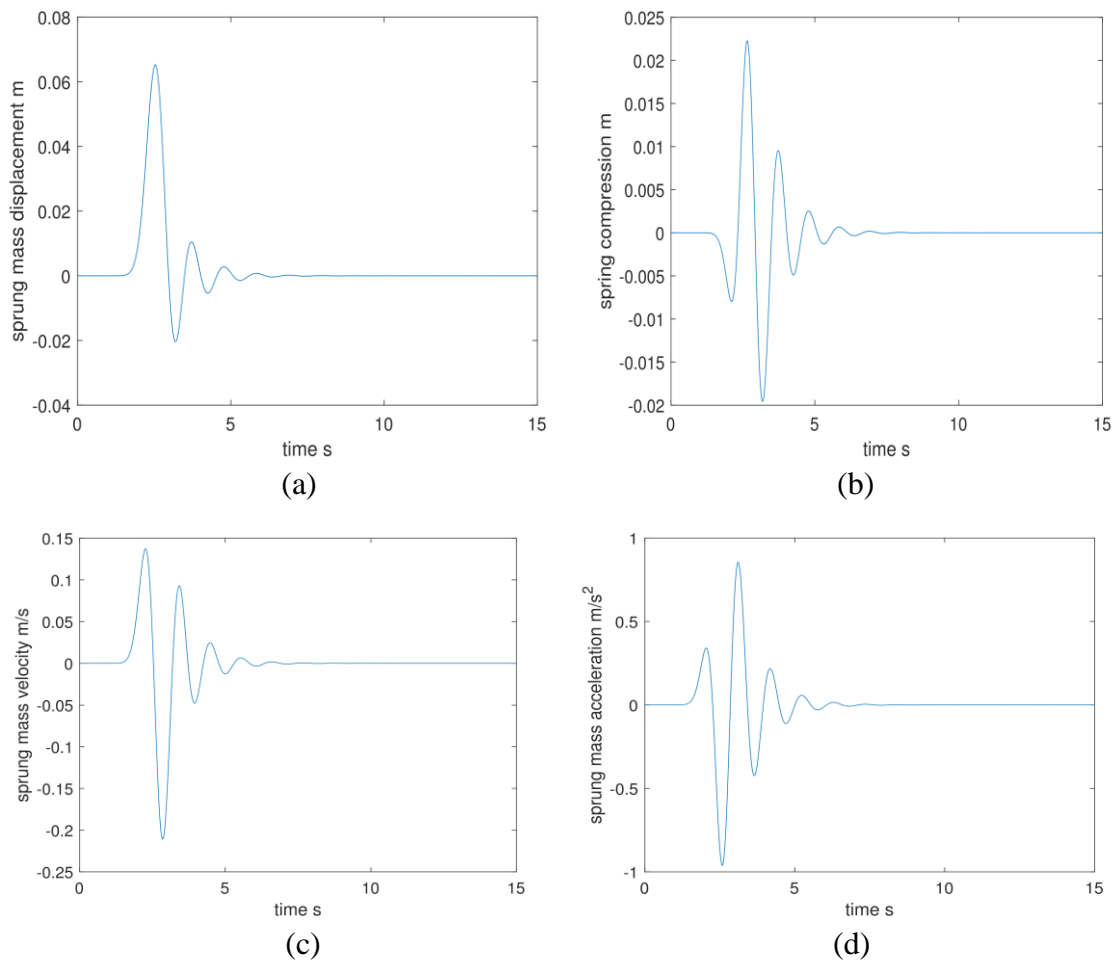


Figure 3. (a), (b), (c) and (d) show the sprung mass displacement, suspension spring compression, sprung mass velocity and sprung mass acceleration with time respectively

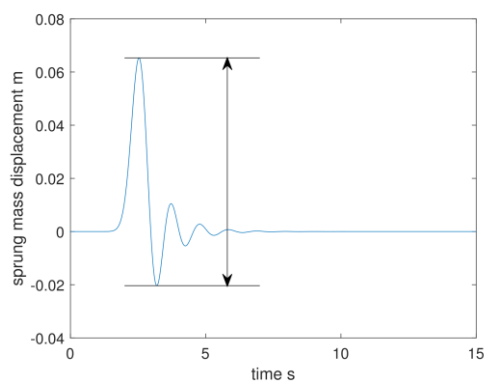


Figure 4. Maximum displacement of the sprung mass as considered for ride comfort

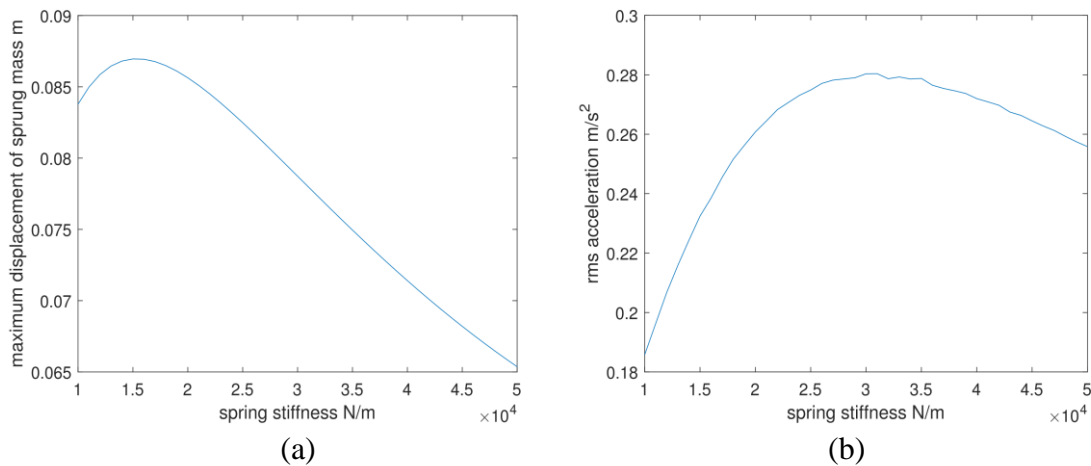


Figure 5. (a), (b) show the variation of maximum sprung mass displacement and sprung mass RMS acceleration with KS

Vehicles are usually driven over a bump at lower speeds. Therefore, the analysis is extended over a range of speed from 10 to 20 km/h so that the effect of changing suspension spring stiffness can be viewed over a broad range of speed. The extension of Figure 5 is given in Figure 6 for the mentioned speed range.

From Figure 6, it can be seen that both the maximum displacement of sprung mass and its RMS acceleration increases with driving speed for the complete range of spring stiffness considered i.e., ride comforts deteriorates with speed by both of our judging standards. This explains why a lower driving speed is a preferred choice for driving over bumps. However, this effect is more pronounced for lower spring stiffness for maximum sprung mass displacement and for higher spring stiffness in case of RMS acceleration.

Figure 6 (a) shows that going for a higher spring stiffness results in a decrease of maximum sprung mass displacement. This is true for all speeds under consideration, however this effect is stronger at lower speeds. In Figure 6 (b) it can be seen that RMS acceleration rises with spring stiffness. The variation of RMS acceleration with spring stiffness is not as sharp as the variation of maximum displacement of sprung mass. Unlike the case of maximum sprung mass displacement, variation of RMS acceleration is weaker at lower speeds.

Considering such results, it can be seen that choosing a higher spring stiffness can be beneficial except for speeds higher than 18 km/h, while for lower speeds a higher k_s will perform better.

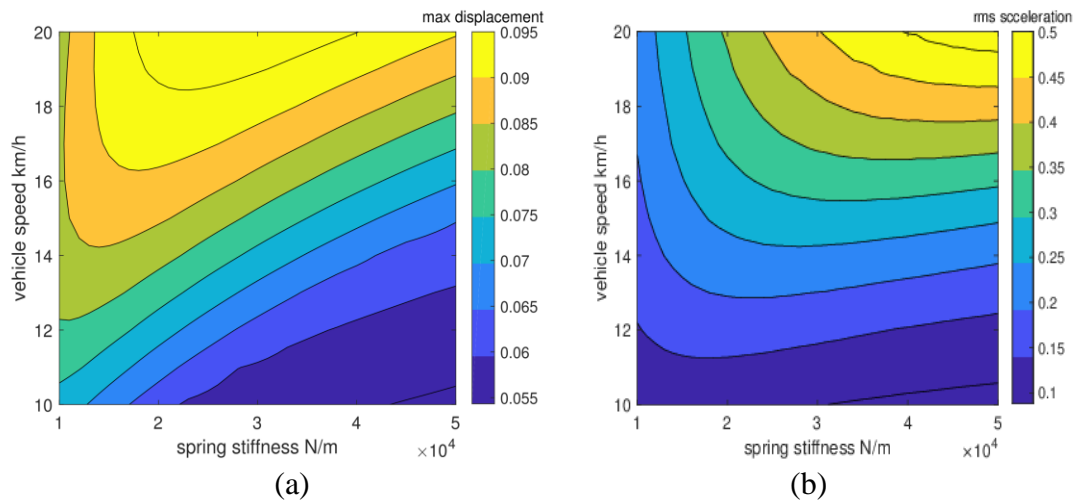


Figure 6. (a), (b) show the variation of maximum sprung mass displacement and sprung mass RMS acceleration with k_s and driving speed

4. Conclusion

The behavior of a quarter car model passing over a Gaussian bump is studied. In this study it has been considered that for a better ride comfort both the RMS value of sprung mass acceleration and its maximum displacement should be as minimum as possible. The effect of suspension spring stiffness on these two parameters is studied in detail for a range of speed from 10 to 20 km/h. It has been observed that increasing the spring stiffness value can enhance the ride comfort over a considerable fraction of the speed range, which is limited to lower than 18 km/hr. The sensitivity of the maximum displacement of sprung mass to suspension spring stiffness is greater as compared to the sensitivity of RMS acceleration. The paper clearly shows that the passenger ride comfort can be improved by utilizing the spring with higher stiffness values and can be used in the design of the passive vehicle suspension systems. This study has been limited only to lower speeds and used a quarter car model. It is planned to further continue the study to include the higher speed range and use a more realistic half car model to explore the effect of changing spring stiffness on ride comfort. The effect of damper tuning can also be studied along with the spring stiffness for having better ride comfort and road holding.

References

- Cho, M. D., Win, H. H., & Latt, A. K. (2017). Vibrational analysis of passive suspension system for solar vehicle. *International Journal of Scientific and Research Publications*, 7(12), 2250-3153.
- Dharankar, C. S., Hada, M. K., & Chandel, S. (2018). Performance improvement of passive suspension of vehicles using position dependent damping. *International Journal of Vehicle Performance*, 4(1), 89-111.
- Dormand, J. R., & Prince, P. J. (1980). A family of embedded runge-kutta formulae. *Journal of Computational and Applied Mathematics*, 6(1), 19-26.
- Hassaan, G. A. (2014). Car dynamics using quarter model and passive suspension, part I: effect of suspension damping and car speed. *International Journal of Computer Techniques*, 1(2), 1-9.

Hassaan, G. A. (2015). Car dynamics using quarter model and passive suspension, part IV: destructive miniature humps (Bumps). *Global Journal of Advanced Research*, 2(2), 451-463.

Hassaan, G. A. (2015). Car dynamics using quarter model and passive suspension, Part II: a novel simple harmonic hump. *Journal of Mechanical and Civil Engineering*, 12(1), 93-100.

Jamali, M. S., Ismail, K. A., Taha, Z., & Aiman, M. F. (2017, October). Development of matlab simulink model for dynamics analysis of passive suspension system for lightweight vehicle. In *Journal of Physics: Conference Series*, IOP Publishing, 908(1), 012066.

Marzbanrad, J., Mohammadi, M., & Mostaani, S. (2013). Optimization of a passive vehicle suspension system for ride comfort enhancement with different speeds based on design of experiment method (DOE) method. *Journal of Mechanical Engineering Research*, 5(3), 50-59.

Phalke, T. P., & Mitra, A. C. (2017). Analysis of ride comfort and road holding of quarter car model by simulink. *Materials Today: Proceedings*, 4(2), 2425-2430.

Satyanarayana, V. S. V., Sateesh, B., & Rao, N. M. (2018). Parameters optimisation of vehicle suspension system for better ride comfort. *International Journal of Vehicle Performance*, 4(2), 186-199.