# Advanced Intuitionistic Fuzzy Weighted Geometric Aggregation Operator for the Intuitionistic Fuzzy Numbers and Its Application to Multi-attribute Decision-making

Devraj Singh[1], Vinit Kumar Sharma[1]
and Ashu Redhu[2,*]

[1]*Faculty of Applied Sciences, Shri Venkateshwara University, Gajraula, U.P., India*
[2]*Department of Mathematics, Amity University Haryana, Gurugram, India*
*E-mail: devrajsingh77777777@gmail.com; vksharmaraj@gmail.com; ashuredhu17@gmail.com*
[*]*Corresponding Author*

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## Abstract

In complex and uncertain scenarios, multi-attribute decision-making (MADM) presents a significant challenge, especially when existing MADM approaches fail to distinguish among the ranking orders (ROs) of alternatives. An important tool to address such challenges is the use of aggregation operators (AOs), which integrate multiple input values into a single representative output. Therefore, in this study, we introduce new operational laws for intuitionistic fuzzy numbers (IFNs) and propose an advanced intuitionistic fuzzy weighted geometric (AIFWG) AO for aggregating IFNs. We also investigate essential properties of the proposed AIFWG AO, such as idempotency, monotonicity, and boundedness. These properties confirm the reliability of

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the proposed AIFWG AO, making it well-suited for real-life decision-making applications. Building on this, we present a new MADM approach under the IFN framework using the proposed AIFWG AO. To validate the effectiveness and robustness of the proposed MADM approach, we solve three MADM problems. The outcomes clearly demonstrate that our method not only addresses the shortcomings of existing MADM approaches but also provides a more reliable ranking of alternatives in uncertain situations.

**Keywords:** Aggregating operator, IFNs, ranking order, MADM.

## 1 Introduction

Multi-attribute decision-making (MADM) addresses difficult situations by considering numerous attributes. It is crucial in real-life decision-making, particularly when there are significant repercussions or several options to consider. This article explains how MADM may improve decision-making for individuals and organizations. MADM can be used in several disciplines, including business, engineering, economics, healthcare, and politics. In business, MADM is used to select the best supplier, hire workers, make lucrative investments, and determine the best marketing approach. In engineering, MADM is used to optimize design, material selection, and system performance. In healthcare, MADM evaluates treatment effectiveness, allocates resources, and assesses quality. In politics, MADM is utilized to make policy decisions, prioritize initiatives, and distribute resources.

However, uncertainty is an unavoidable aspect of decision-making process. To deal with this, several approaches have been developed, one of these is the famous Fuzzy Set Theory (FST) introduced by Zadeh [33] in 1965, which has become quite well-known. Fuzzy Sets (FS) have opened up new ways of making decisions, giving us a more nuanced and better option than traditional methods when faced with the mystery of ambiguity. By incorporating membership grade (MG) rather than binary classification, FST provides a powerful way to model uncertainty and vagueness. FST's impact can be felt in many areas of academia, as it brings new life to several fields. Also, a lot of other works were made using Fuzzy Set extensions.

In 1986, Atanassov [2] introduced the notion of intuitionistic fuzzy sets (IFSs), which alongwith a MG also incorporate a non-membership grade (NMG) satisfying the condition $0 \leq MG + NMG \leq 1$, where $MG, NMG \in [0, 1]$. Compared to traditional FSs, IFSs provide enhanced flexibility in modeling uncertain information. Since then, numerous researchers have widely utilized the notion of IFSs in various decision-making applications [3–5, 7, 9, 10, 15–18, 26, 28, 31, 32, 34]. Krishankumar et al. [15] proposed entropy measure for IFSs and applied it to select the cloud vendor. Garg et al. [9] proposed distance measure and based on it, developed a decision-making model within the context of IFSs. Patel et al. [18] introduced similarity measures for IFSs and applied them to face-recognition and software quality assessment. Augustine [3] developed correlation coefficient for IFSs and applied it solve MADM problems. Thao and Chou [26] proposed entropy measure and similarity measure for IFSs and applied them to evaluate software quality. Dhankhar and Kumar [5] developed an MADM approach based on the proposed possibility degree measure for IFNs. Mahanta and Panda [17] proposed a distance measure for IFSs and used it to solve various decision-making problems. Zou et al. [34] developed improved IF weighted geometric AOs within the context of IFNs. Garg and Kumar [7] presented an improved possibility degree measure for IFNs and employed it develop an MADM approach. Patel et al. [19] proposed similarity measure for IFSs and presented an image fusion approach. Thao et al. [27] developed a distance measure for IFSs using score function and proposed a MADM approach based on the proposed distance measure.

Aggregation operator (AO) is an important aspect of solving MADM problems. It is a mathematical tool used to aggregate multiple preference values into single value. In the area of AOs, a lot of work has been done by researchers [1, 8, 11, 14, 20–24, 30]. Senapati et al. [24] presented new operational rules for IFNs and weighted AOs based on Aczel-Alsina t-norm and t-conorm. Seikh and Mandal [23] proposed IF AOs based on Dombi norms and used them to solve MADM problems. Alcantud [1] introduced IF weighted geometric AOs and utilized them to solve group decision-making problems. Rahman et al. [20] presented logarithmic AOs under the IFN environment. Khan et al. [14] proposed IF power AO based on Schweizer-Sklar norms within the context of IFN environment. Unver [29] proposed weighted arithmetic and geometric AOs based on defined Gaussian norms under the context of IFNs. Hussain et al. [12] developed prioritized geometric and weighted prioritized geometric AOs based on Sugeno-Weber norms and proposed a decision making approach to identify the best digital security method. Sharma et al. [25] introduced power arithmetic and weighted power arithmetic AOs based on Einstein norms and developed a MADM approach based on them within the IF environment. Hussain et al. [13] proposed AOs based on Hamy mean and Aczel-Alsina norms and developed a decision making model based on them to evaluate the construction material.

### 1.1 Research Gaps and Motivations of This Study

The research gaps identified in the literature and the underlying motivations for this study are outlined as follows:

(i) Several existing AOs used to handle the intuitionistic fuzzy information fails to effectively capture uncertainty and provide less accurate decision outcomes. Thus, there is a need to develop more flexible AO which ensures reliable and robust decision making outcomes.
(ii) In this study, we observed that the MADM approaches proposed by Garg and Kumar [7] and Zou et al. [34] exhibit limitations, particularly in their inability to differentiate the ranking orders of alternatives under certain conditions. Therefore, it is essential to develop a new MADM approach that overcomes these shortcomings presented in the MADM approaches of Garg and Kumar [7] and Zou et al. [34] and provide reliable results.

### 1.2 Contributions of This Study

The main contributions of this study are outlined as follows:

(i) We present new operational laws for IFNs including, multiplication operation and scalar power operation.
(ii) We introduce an advanced intuitionistic fuzzy weighted geometric (AIFWG) AO to aggregate the information. We also examine key desirable properties of the proposed AIFWG AO, such as idempotency, monotonicity and boundedness.
(iii) We present a novel MADM approach for the IFNs environment by using the proposed AIFWG AO.
(iv) We present a comparative analysis to highlight the strengths of the proposed MADM against existing MADM approaches given in [7, 34]. The proposed MADM approach is highly effective and applicable approach for addressing the MADM problems within the environment of IFNs.

To achieve the above objectives, this paper is organized in the following manner: Section 2 covers the preliminaries relevant to this study. In Section 3, we present new operational laws for intuitionistic fuzzy numbers (IFNs) and develop an advanced intuitionistic fuzzy weighted geometric (AIFWG) aggregation operator. Section 4 introduces a novel MADM approach based on the proposed AIFWG operator for IFNs. Section 5 provides illustrative examples to demonstrate the proposed MADM approach and highlights its advantages compared to existing MADM approaches. Finally, Section 6 highlights major findings and suggests future study directions.

## 2 Preliminaries

**Definition 1 [2].** In universal set $X$, an IFS $I_F$ is represented by

$$I_F = \{\langle x, \eta(x), \upsilon(x)\rangle | x \in X\}$$

where $\eta(x), \upsilon(x) \in [0,1]$, represents MG and NMG of $x$ to $I_F$, respectively, such that $0 \leq \eta(x) + \upsilon(x) \leq 1$ holds, and in turn, the hesitance of $x$ to $I_F$ is defined as $\pi(x) = 1 - \eta(x) - \upsilon(x)$, where $0 \leq \pi(x) \leq 1, x \in X$. Usually, the pair $\langle \eta, \upsilon\rangle$ is called an IFN.

**Definition 2 [2].** For comparing two IFNs $\Phi_1 = \langle \eta_1, \upsilon_1\rangle$ and $\Phi_2 = \langle \eta_2, \upsilon_2\rangle$ the operating rules are given as:

(i) $\Phi_1 \succeq \Phi_2 \Leftrightarrow \eta_1 \geq \eta_2$ and $\upsilon_1 \leq \upsilon_2$;
(ii) $\Phi_1 = \Phi_2 \Leftrightarrow \eta_1 = \eta_2$ and $\upsilon_1 = \upsilon_2$.

**Definition 3 [6].** For the IFNs $\Phi_1 = \langle \eta_1, \upsilon_1\rangle, \Phi_2 = \langle \eta_2, \upsilon_2\rangle \ldots, \ldots,$ and $\Phi_n = \langle \eta_n, \upsilon_n\rangle$, the aggregated value by using the intuitionistic fuzzy Einstein weighted geometric interactive averaging (IFEWGIA) AO is given as follows:

$$IFEWGIA(\Phi_1, \Phi_2, \ldots \Phi_n) = \left\langle \begin{array}{l} \dfrac{2\left\{\prod_{t=1}^{n}(1-\upsilon_t)^{\varphi_t} - \prod_{t=1}^{n}(1-\eta_t-\upsilon_t)^{\varphi_t}\right\}}{\prod_{t=1}^{n}(1+\upsilon_t)^{\varphi_t} + \prod_{t=1}^{n}(1-\upsilon_t)^{\varphi_t}}, \\ \dfrac{\prod_{t=1}^{n}(1+\upsilon_t)^{\varphi_t} - \prod_{t=1}^{n}(1-\upsilon_t)^{\varphi_t}}{\prod_{t=1}^{n}(1+\upsilon_t)^{\varphi_t} + \prod_{t=1}^{n}(1-\upsilon_t)^{\varphi_t}} \end{array} \right\rangle. \tag{1}$$

where $\varphi_t$ denotes the weight of the IFN $\Phi_t, \varphi_t \in [0,1], \sum_{t=1}^{n} \varphi_t = 1$, and $t = 1, 2, \ldots, n$.

**Definition 4 [34].** For the IFNs $\Phi_1 = \langle \eta_1, \upsilon_1\rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2\rangle \ldots, \ldots,$ and $\Phi_n = \langle \eta_n, \upsilon_n\rangle$, the aggregated value by using the improved intuitionistic fuzzy weighted geometric (IIFWG) AO is given as follows:

$$IIFWG(\Phi_1, \Phi_2, \ldots \Phi_n) = \left\langle \begin{array}{l} 1 - \dfrac{1}{\lambda}\left(1 - \prod\limits_{t=1}^{n}(1-\lambda(1-\eta_t))^{\varphi_t}\right), \\ 1 - \dfrac{1}{\lambda}\left(1 - \prod\limits_{t=1}^{n}(1-\lambda(1-\upsilon_t))^{\varphi_t}\right) \end{array} \right\rangle \tag{2}$$

where $\varphi_t$ denotes the weight of the IFN $\Phi_t, \varphi_t \in [0,1], \sum_{t=1}^{n} \varphi_t = 1, t = 1, 2, \ldots, n$ and $0 < \lambda < 1$.

**Definition 5 [7].** Consider $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$ and $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle$ be two IFNs, then the possibility degree measure for comparing $\Phi_1$ and $\Phi_2$ is defined as:

(i)
$$P(\Phi_1 \succeq \Phi_2) = \min\left(\max\left(\frac{1+\eta_1-2\eta_2-\upsilon_2}{\pi_1+\pi_2}, 0\right), 1\right), \quad (3)$$
where, either $\pi_1 \neq 0$ or $\pi_2 \neq 0$.

(ii) If $\pi_1 = \pi_2 = 0$, then
$$P(\Phi_1 \succeq \Phi_2) = \begin{cases} 1: & \eta_1 > \eta_2 \\ 0: & \eta_1 < \eta_2 \\ 0.5: & \eta_1 = \eta_2 \end{cases}. \quad (4)$$

## 3 Advanced Intuitionistic Fuzzy Weighted Geometric Aggregation Operator

In this section, we introduce the advanced intuitionistic fuzzy weighted geometric (AIFWG) aggregation operator (AO) to aggregate the intuitionistic fuzzy numbers (IFNs).

**Definition 6.** Let $\Phi = \langle \eta, \upsilon \rangle$, $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle, \ldots$ and $\Phi_n = \langle \eta_n, \upsilon_n \rangle$ be IFNs. The operation laws proposed for these IFNs are outlined below:

(i) $\Phi_1 \otimes \Phi_2 \otimes \cdots \otimes \Phi_n = \langle 1 - \frac{1}{\epsilon}(1 - \prod_{t=1}^{n}(1-\epsilon(1-\eta_t))), \frac{1}{\epsilon}(1 - \prod_{t=1}^{n}(1-\epsilon\upsilon_t))\rangle$

(ii) $\Phi^{\kappa} = \langle 1 - \frac{1}{\epsilon}(1-(1-\epsilon(1-\eta_t))^{\kappa}), \frac{1}{\epsilon}(1-(1-\epsilon\upsilon_t)^{\kappa})\rangle$

where $\kappa > 0$ and $0 < \epsilon < 1$.

**Definition 7.** The proposed AIFWG operator for aggregating the IFNs $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle \ldots, \ldots$, and $\Phi_n = \langle \eta_n, \upsilon_n \rangle$ is shown as follows:

$$\begin{aligned} AIFWG(\Phi_1, \Phi_2, \ldots \Phi_n) &= \bigotimes_{t=1}^{n} \Phi_t^{\varphi_t} \\ &= \Phi_1^{\varphi_1} \bigotimes \Phi_2^{\varphi_2} \bigotimes \ldots \bigotimes \Phi_n^{\varphi_n} \end{aligned} \quad (5)$$

where $\varphi_t$ represents the weight of IFN $\Phi_t, \varphi_t \in [0,1], \sum_{t=1}^{n} \varphi_t = 1, t = 1, 2, \ldots, n$.

**Theorem 1.** For the IFNs $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle \ldots, \ldots$, and $\Phi_n = \langle \eta_n, \upsilon_n \rangle$, the aggregated value by using the proposed AIFWG AO is an IFN and given as follows:

$$AIFWG(\Phi_1, \Phi_2, \ldots \Phi_n) = \left\langle \begin{array}{l} 1 - \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon(1-\eta_t))^{\varphi_t}\right), \\ \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon\upsilon_t)^{\varphi_t}\right) \end{array} \right\rangle \quad (6)$$

where $\varphi_t$ represents the weight of IFN $\Phi_t, \varphi_t \in [0,1], \sum_{t=1}^{n} \varphi_t = 1, t = 1, 2, \ldots, n$ and $0 < \epsilon < 1$. In this study, we take $\epsilon = 0.99$ for the proposed AIFWG operator stated in Equation (6).

**Proof.** Let $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle \ldots, \ldots$, and $\Phi_n = \langle \eta_n, \upsilon_n \rangle$ be $n$ IFVs. By using the proposed operating rules given in Definition 6, for $t = 1, 2, \ldots, n$, we have

$$\Phi_t^{\varphi_t} = \left\langle 1 - \frac{1}{\epsilon}(1 - (1 - \epsilon(1-\eta_t))^{\varphi_t}), \frac{1}{\epsilon}(1 - (1 - \epsilon\upsilon_t)^{\varphi_t}) \right\rangle,$$

$$\bigotimes_{t=1}^{n} \Phi_t^{\varphi_t} = \left\langle 1 - \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon(1-\eta_t))^{\varphi_t}\right), \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon\upsilon_t)^{\varphi_t}\right) \right\rangle.$$

Hence, by using Equation (5), we have

$$AIFWG(\Phi_1, \Phi_2, \ldots \Phi_n) = \bigotimes_{t=1}^{n} \Phi_t^{\varphi_t},$$

$$AIFWG(\Phi_1, \Phi_2, \ldots \Phi_n) = \left\langle \begin{array}{l} 1 - \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon(1-\eta_t))^{\varphi_t}\right), \\ \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon\upsilon_t)^{\varphi_t}\right) \end{array} \right\rangle.$$

Let $\eta = 1-\frac{1}{\epsilon}(1-\prod_{t=1}^{n}(1-\epsilon(1-\eta_t))^{\varphi_t})$ and $\upsilon = \frac{1}{\epsilon}(1-\prod_{t=1}^{n}(1-\epsilon\upsilon_t)^{\varphi_t})$. We must show that $\eta$ and $\upsilon$ meet the following attribute:

(i) $0 \leq \eta \leq 1$ and $0 \leq \upsilon \leq 1$,
(ii) $0 \leq \eta + \upsilon \leq 1$.

First, we prove that $0 \leq \eta \leq 1$. Since $\Phi_1 = \langle \eta_1, \upsilon_1 \rangle$, $\Phi_2 = \langle \eta_2, \upsilon_2 \rangle \ldots, \ldots$, and $\Phi_n = \langle \eta_n, \upsilon_n \rangle$ are the IFNs, we get $0 \leq \eta_t \leq 1$, $0 \leq \upsilon_t \leq 1$ and $0 \leq \eta_t + \upsilon_t \leq 1$ for all $t = 1, 2, \ldots, n$. It implies that $0 \leq (1-\eta_t) \leq 1$ for all $t = 1, 2, \ldots, n$. Since $0 \leq \epsilon \leq 1$, $0 \leq \varphi_t \leq 1$ and $\sum_{t=1}^{n} \varphi_t = 1$, we get $0 \leq (1-\epsilon(1-\eta_t))^{\varphi_t} \leq 1$ for all $t = 1, 2, \ldots, n$. It implies that $0 \leq \prod_{t=1}^{n}(1-\epsilon(1-\eta_t))^{\varphi_t} \leq 1$. Hence, $0 \leq \eta \leq 1$. Similarly, we can prove that $0 \leq \upsilon \leq 1$. Now, we prove that $0 \leq \eta + \upsilon \leq 1$. We have

$$\begin{aligned} \eta + \upsilon &= 1 - \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1-\epsilon(1-\eta_t))^{\varphi_t}\right) + \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1-\epsilon\upsilon_t)^{\varphi_t}\right) \\ &= 1 - \frac{1}{\epsilon}\left(\prod_{t=1}^{n}(1-\epsilon\upsilon_t)^{\varphi_t} - \prod_{t=1}^{n}(1-\epsilon(1-\eta_t))^{\varphi_t}\right) \\ &\leq 1. \end{aligned}$$

Since $\eta \geq 0$ and $\upsilon \geq 0$, we get $\eta + \upsilon \geq 0$. Hence, $0 \leq \eta + \upsilon \leq 1$.

**Example 1.** Let $\Phi_1 = \langle 0.3, 0.5 \rangle$, $\Phi_2 = \langle 0.5, 0.4 \rangle$, and $\Phi_3 = \langle 0.2, 0.7 \rangle$ be three IFNs with corresponding weights $\varphi_1 = 0.4, \varphi_2 = 0.2$, and $\varphi_3 = 0.4$. The aggregated value of these IFNs, obtained using Equation (6), is

$$\begin{aligned} AIFWG(\Phi_1, \Phi_2, \Phi_3) &= \left\langle \begin{array}{l} 1 - \frac{1}{0.99}\left(1 - \begin{pmatrix} (1-0.99(1-0.3))^{0.4}. \\ (1-0.99(1-0.5))^{0.2}. \\ (1-0.99(1-0.2))^{0.4} \end{pmatrix}\right), \\ \frac{1}{0.99}\left(1 - \begin{pmatrix} (1-0.99\times 0.5)^{0.4}. \\ (1-0.99\times 0.4)^{0.2}. \\ (1-0.99\times 0.7)^{0.4} \end{pmatrix}\right) \end{array} \right\rangle \\ &= \langle 0.2831, 0.5768 \rangle. \end{aligned}$$

**Property 1 (*Idempotency*).** Let $\Phi_1, \Phi_2, \ldots$ and $\Phi_{\mathrm{n}}$ be IFNs and let the weights of the IFNs $\Phi_1, \Phi_2, \ldots$ and $\Phi_{\mathrm{n}}$ be $\varphi_1, \varphi_2, \ldots$ and $\varphi_n$, respectively, where $\varphi_t \in [0,1]$, $\sum_{t=1}^{n} \varphi_t = 1$ and $\forall\, t = 1, 2, \ldots, n$. If $\Phi_1 = \Phi_2, \ldots = \Phi_n = \Phi$, then

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) = \Phi$$

**Proof.** Given that the IFNs $\Phi_1, \Phi_2, \ldots$ and $\Phi_n$ have corresponding weights $\varphi_1, \varphi_2, \ldots$ and $\varphi_n$, respectively, where each $\varphi_t \in [0, 1]$, $\sum_{t=1}^{n} \varphi_t = 1$ and $t = 1, 2, \ldots, n$. If $\Phi_1 = \Phi_2, \ldots = \Phi_n = \Phi$, then based on the proposed AIFWG operator stated in Equation (5), we have

$$\begin{aligned} AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) &= \Phi_1^{\varphi_1} \bigotimes \Phi_2^{\varphi_2} \bigotimes \cdots \bigotimes \Phi_n^{\varphi_n} \\ &= \Phi^{\varphi_1} \bigotimes \Phi^{\varphi_2} \bigotimes \cdots \bigotimes \Phi^{\varphi_n} \\ &= \Phi^{\varphi_1 + \varphi_2 + \cdots + \varphi_3} \\ &= \Phi. \end{aligned}$$

**Property 2 (*Boundedness*).** Let $\Phi_1, \Phi_2, \ldots$ and $\Phi_n$ be IFNs, let $\Phi^- = min\{\Phi_1, \Phi_2, \ldots, \Phi_n\}$ and let $\Phi^+ = max\{\Phi_1, \Phi_2, \ldots, \Phi_n\}$. Then,

$$\Phi^- \leq AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) \leq \Phi^+.$$

**Proof.** Since $\Phi^- = min\{\Phi_1, \Phi_2, \ldots, \Phi_n\}$ and $\Phi^+ = max\{\Phi_1, \Phi_2, \ldots, \Phi_n\}$, then by using the proposed AIFWG operator given in Equation (5), we obtain

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) = \bigotimes_{t=1}^{n} \Phi_t^{\varphi_t} \leq \bigotimes_{t=1}^{n} (\Phi_t^+)^{\varphi_t} = (\Phi^+)^{\sum_{t=1}^{n} \varphi_t},$$

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) = \bigotimes_{t=1}^{n} \Phi_t^{\varphi_t} \geq \bigotimes_{t=1}^{n} (\Phi_t^-)^{\varphi_t} = (\Phi^-)^{\sum_{t=1}^{n} \varphi_t}.$$

Because $\sum_{t=1}^{n} \varphi_t = 1$, we get $\Phi^- \leq$ ***AIFWG***$(\Phi_1, \Phi_2, \ldots, \Phi_n) \leq \Phi^+$.

**Property 3 (*Monotonicity*).** Let $\Phi_1, \Phi_2, \ldots, \Phi_n, \Phi_1^*, \Phi_2^*, \ldots,$ and $\Phi_n^*$ be IFNs. If $\Phi_t \leq \Phi_t^*$, where $t$= $1, 2, \ldots$,$n$, then

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) \leq AIFWG(\Phi_1^*, \Phi_2^*, \ldots, \Phi_n^*).$$

**Proof.** Based on the proposed AIFWG AO given in Equation (5), we have

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) = \bigotimes_{t=1}^{n} \Phi_t^{\varphi_t},$$

$$AIFWG(\Phi_1^*, \Phi_2^*, \ldots, \Phi_n^*) = \bigotimes_{t=1}^{n} \Phi_t^{*\varphi_t}$$

Because $\Phi_t \leq \Phi_t^*$, where $t = 1, 2, \ldots, n$, we get $\bigotimes_{t=1}^{n} \Phi_t^{\varphi_t} \leq \bigotimes_{t=1}^{n} \Phi_t^{*\varphi_t}$. Therefore, we obtain

$$AIFWG(\Phi_1, \Phi_2, \ldots, \Phi_n) \leq AIFWG(\Phi_1^*, \Phi_2^*, \ldots, \Phi_n^*).$$

## 4 A Novel MADM Approach Based on the Proposed AIFWG Aggregation Operator (AO) of IFNs

In the following, we introduce a novel MADM approach utilizing the proposed AIFWG AO of IFNs. Let $\Xi_1, \Xi_2, \ldots, \Xi_m$ be $m$ alternatives and let $\Lambda_1, \Lambda_2, \ldots, \Lambda_n$ be $n$ attributes with their corresponding weights $\varphi_1, \varphi_2, \ldots, \varphi_n$, where $\varphi_t \in [0, 1]$ and $\sum_{t=1}^{n} \varphi_t = 1$. The decision-maker assesses the alternative $\Xi_s$ with respect to the attribute $\Lambda_t$ using a IFN $\widetilde{\Phi}_{st} = \langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle$ to form a decision matrix $\tilde{D} = (\widetilde{\Phi}_{st})_{m \times n}$, given as follows:

$$\tilde{D} = \begin{array}{c} \\ \Xi_1 \\ \Xi_2 \\ \vdots \\ \Xi_m \end{array} \begin{array}{c} \begin{array}{cccc} \Lambda_1 & \Lambda_2 & \ldots & \Lambda_n \end{array} \\ \begin{pmatrix} \widetilde{\Phi}_{11} & \widetilde{\Phi}_{12} & \cdots & \widetilde{\Phi}_{1n} \\ \widetilde{\Phi}_{21} & \widetilde{\Phi}_{22} & \ldots & \widetilde{\Phi}_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ \widetilde{\Phi}_{m1} & \widetilde{\Phi}_{m2} & \ldots & \widetilde{\Phi}_{mn} \end{pmatrix} \end{array},$$

The steps involved in the proposed MADM approach are outlined as follows:

**Step 1:** Transform the decision matrix $\tilde{D} = (\widetilde{\Phi}_{st})_{m \times n} = (\langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle)_{m \times n}$ into the normalized decision matrix (NDMx) $D = (\Phi_{st})_{m \times n} = (\langle \eta_{st}, \upsilon_{st} \rangle)_{m \times n}$, as defined below:

$$\Phi_{st} = \begin{cases} \langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle: & \text{for benefit type attribute} \\ \langle \widetilde{\upsilon}_{st}, \widetilde{\eta}_{st} \rangle: & \text{for cost type attribute,} \end{cases} \tag{7}$$

where, $s = 1, 2, \ldots, m$ and $t = 1, 2, \ldots, n$.

**Step 2:** Using the proposed AIFWG AO defined in Equation (6), we aggregate the IFNs $\Phi_{s1}, \Phi_{s2}, \ldots, \Phi_{sn}$ from the $s$th row of the NDMx $D = (\Phi_{st})_{m \times n}$ to obtain the overall aggregated IFN $\Phi_s = \langle \eta_s, \upsilon_s \rangle$, expressed as:

$$\begin{aligned} \Phi_s &= \langle \eta_s, \upsilon_s \rangle \\ &= AIFWG(\Phi_{s1}, \Phi_{s2}, \ldots, \Phi_{sn}) \end{aligned}$$

$$= \left\langle 1 - \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon(1 - \eta_t))^{\varphi_t}\right), \right.$$
$$\left. \frac{1}{\epsilon}\left(1 - \prod_{t=1}^{n}(1 - \epsilon\upsilon_t)^{\varphi_t}\right)\right\rangle. \tag{8}$$

**Step 3:** Compute the score value $S(\Phi_s)$ for the obtained IFN $\Phi_s = \langle \eta_s, \upsilon_s \rangle$ corresponding to the alternative $\Xi_s$, as follows:

$$S(\Phi_\mathrm{s}) = \frac{1}{3}(2\eta_\mathrm{s} - \upsilon_\mathrm{s}(1 + \pi_\mathrm{s}) + 1), \tag{9}$$

where, $\pi_s = 1 - \eta_s - \upsilon_s$.

**Step 4:** Arrange the obtained score values in descending order to determine the ranking order (RO) of the alternatives $\Xi_s (s = 1, 2, \ldots, m)$, and select the best alternative.

Figure 1 represents comprehensive flow chart of the proposed MADM method.

## 5 Illustrative Examples of Proposed MADM Approach

**Example 2 [7].** With the rising population and infrastructure, New Delhi faces severe traffic congestion, especially during peak hours. To address this, the New Delhi Development Authority (NDDA) plans to build a flyover at a busy intersection and has issued a global tender to select the best contractor. The evaluation is based on five attributes: project cost $(\Lambda_1)$, completion time $(\Lambda_2)$, technical capability $(\Lambda_3)$, financial status $(\Lambda_4)$, and company background $(\Lambda_5)$, with corresponding weights $\varphi_1 = 0.3, \varphi_2 = 0.25$, $\varphi_3 = 0.1, \varphi_4 = 0.15$, and $\varphi_5 = 0.2$. Four companies: PNC Infratech Ltd. $(\Xi_1)$, Hindustan Construction Company $(\Xi_2)$, J.P. Construction $(\Xi_3)$, and Gammon India Ltd. $(\Xi_4)$ have submitted bids, and decision maker assess them under an IFS environment by using an IFN $\widetilde{\Phi}_{st} = \langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle$ to form the decision matrix $\tilde{D} = (\widetilde{\Phi}_{st})_{m \times n}$, given as follows:

$$\tilde{D} = \begin{array}{c} \\ \Xi_1 \\ \Xi_2 \\ \Xi_3 \\ \Xi_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \quad\Lambda_1 & \quad\quad\Lambda_2 & \quad\quad\Lambda_3 & \quad\quad\Lambda_4 & \quad\quad\Lambda_5 \end{array} \\ \begin{pmatrix} \langle 0.3, 0.6 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.5, 0.2 \rangle & \langle 0.7, 0.1 \rangle \\ \langle 0.5, 0.3 \rangle & \langle 0.6, 0.2 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.4, 0.2 \rangle \\ \langle 0.5, 0.4 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.6, 0.2 \rangle & \langle 0.5, 0.3 \rangle \\ \langle 0.6, 0.2 \rangle & \langle 0.4, 0.3 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.4, 0.4 \rangle & \langle 0.2, 0.8 \rangle \end{pmatrix} \end{array}.$$

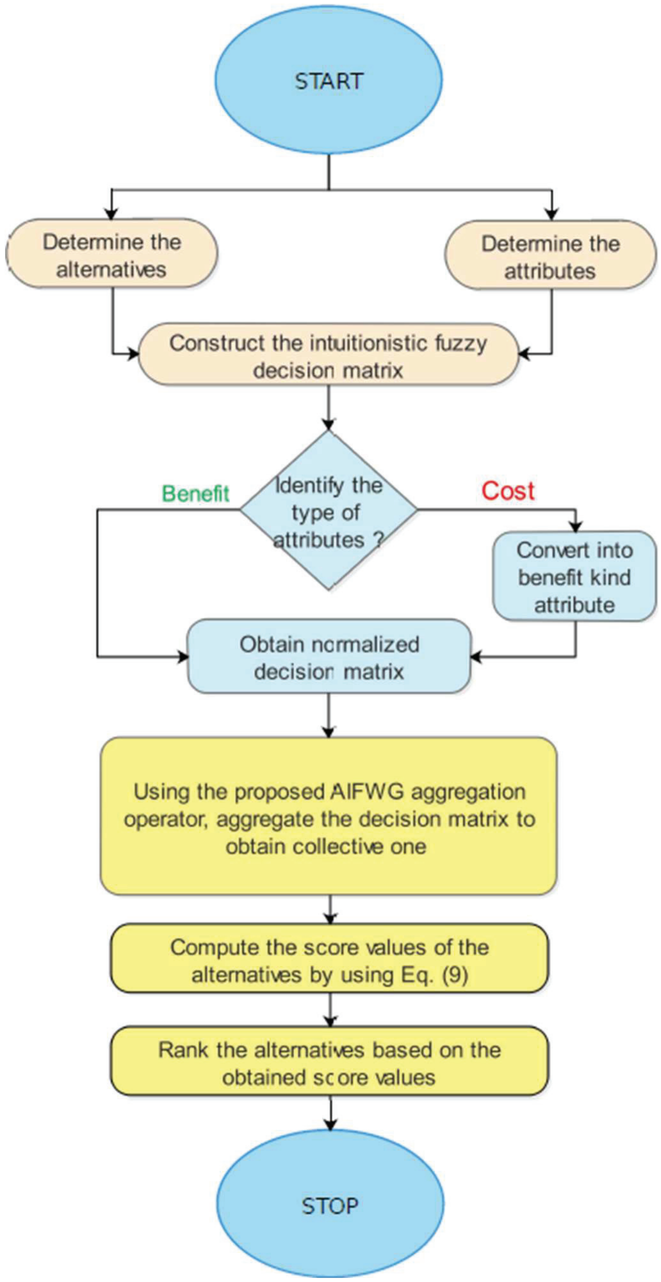


**Figure 1** Flowchart of the proposed MCDM method.

In order to solve this MADM problem, we utilize the proposed MADM approach described in this paper as follows:

**Step 1:** As $\Lambda_1$ and $\Lambda_2$ are cost-type attributes, therefore by using Equation (7), we obtain the NDMx, where

$$\tilde{D} = \begin{array}{c} \\ \Xi_1 \\ \Xi_2 \\ \Xi_3 \\ \Xi_4 \end{array} \begin{array}{c} \begin{array}{ccccc} \Lambda_1 & \Lambda_2 & \Lambda_3 & \Lambda_4 & \Lambda_5 \end{array} \\ \begin{pmatrix} \langle 0.6, 0.3 \rangle & \langle 0.4, 0.5 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.5, 0.2 \rangle & \langle 0.7, 0.1 \rangle \\ \langle 0.3, 0.5 \rangle & \langle 0.2, 0.6 \rangle & \langle 0.5, 0.4 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.4, 0.2 \rangle \\ \langle 0.4, 0.5 \rangle & \langle 0.2, 0.7 \rangle & \langle 0.8, 0.1 \rangle & \langle 0.6, 0.2 \rangle & \langle 0.5, 0.3 \rangle \\ \langle 0.2, 0.6 \rangle & \langle 0.3, 0.4 \rangle & \langle 0.7, 0.2 \rangle & \langle 0.4, 0.4 \rangle & \langle 0.2, 0.8 \rangle \end{pmatrix} \end{array}.$$

**Step 2:** Using the proposed AIFWG AO defined in Equation (8), we obtain the overall aggregated IFN $\Phi_s = \langle \eta_s, \upsilon_s \rangle$ of the alternative $\Xi_s$, where $\Phi_1 = \langle 0.5527, 0.3001 \rangle$, $\Phi_2 = \langle 0.3360, 0.4433 \rangle$, $\Phi_3 = \langle 0.4017, 0.4636 \rangle$, and $\Phi_4 = \langle 0.2791, 0.5599 \rangle$.

**Step 3:** Using Equation (9), we compute the score value $S(\Phi_s)$ corresponding to the alternative $\Xi_s$, where $S(\Phi_1) = 0.5871$, $S(\Phi_2) = 0.3770$, $S(\Phi_3) = 0.4258$, and $S(\Phi_4) = 0.3027$.

**Step 4:** Since, $S(\Phi_1) > S(\Phi_3) > S(\Phi_2) > S(\Phi_4)$, therefore, the RO of alternatives $\Xi_1, \Xi_2, \Xi_3$, and $\Xi_4$ is "$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$". Hence, PNC Infratech Ltd. $(\Xi_1)$ is the best option.

Table 1 and Figure 2 present a comparison of the RO of the alternatives $\Xi_1, \Xi_2, \Xi_3$, and $\Xi_4$ obtained using different MADM approaches for Example 2. It is clear from Table 1 and Figure 2 that Garg and Kumar [7] MADM approach obtains the RO "$\Xi_1 \succ \Xi_4 \succ \Xi_3 \succ \Xi_2$" whereas both Zou et al. [34] MADM approach and the proposed MADM approach obtain the RO "$\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$". The difference in results can be attributed to the ranking methods, where Garg and Kumar [7] MADM approach uses possibility degree measure to rank alternatives, while Zou et al. [34] MADM approach and the proposed MADM approach use score function. Despite

**Table 1** The ROs of the alternatives obtained by different MADM approaches for Example 2

| MADM Approaches | ROs |
|---|---|
| Garg and Kumar [7] MADM approach | $\Xi_1 \succ \Xi_4 \succ \Xi_3 \succ \Xi_2$ |
| Zou et al. [34] MADM approach | $\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$ |
| Proposed MADM approach | $\Xi_1 \succ \Xi_3 \succ \Xi_2 \succ \Xi_4$ |

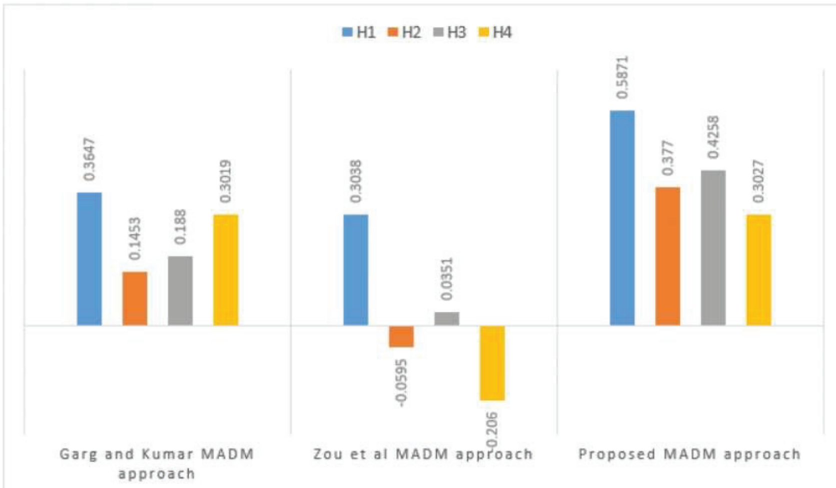


**Figure 2** Graphical comparison of POs obtained by various MADM methods for Example 2.

this difference, the optimal alternative identified by [7, 34] and the proposed method remains $\Xi_1$, confirming the reliability of the proposed MADM approach.

**Example 3.** Consider three alternatives $\Xi_1, \Xi_2$, and $\Xi_3$, and three attributes $\Lambda_1, \Lambda_2$, and $\Lambda_3$ with corresponding weights $\varphi_1 = 0.3, \varphi_2 = 0.4$, and $\varphi_3 = 0.3$. The decision maker wants to assess the alternatives with respect to the attributes under an IFS environment by using an IFN $\widetilde{\Phi}_{st} = \langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle$ to form the decision matrix $\tilde{D} = (\widetilde{\Phi}_{st})_{m \times n}$, given as follows:

$$\tilde{D} = \begin{matrix} & \Lambda_1 & \Lambda_2 & \Lambda_3 \\ \Xi_1 & \langle 0.3, 0.4 \rangle & \langle 0, 1 \rangle & \langle 0.7, 0.2 \rangle \\ \Xi_2 & \langle 0, 1 \rangle & \langle 0.6, 0.3 \rangle & \langle 0.4, 0.2 \rangle \\ \Xi_3 & \langle 0.5, 0.4 \rangle & \langle 0, 1 \rangle & \langle 0.2, 0.6 \rangle \end{matrix}.$$

In order to solve this MADM problem, we utilize the proposed MADM approach described in this paper as outlined below:

**Step 1:** As all the attributes are benefit-type, normalizing the decision matrix is not required.

**Step 2:** Using the proposed AIFWG AO defined in Equation (8), we obtain the overall aggregated IFN $\Phi_s = \langle \eta_s, \upsilon_s \rangle$ of the alternative $\Xi_s$, where $\Phi_1 = \langle 0.0910, 0.8813 \rangle$, $\Phi_2 = \langle 0.1481, 0.8038 \rangle$, and $\Phi_3 = \langle 0.0713, 0.9051 \rangle$.

**Step 3:** Using Equation (9), we compute the score value $S(\Phi_s)$ corresponding to the alternative $\Xi_s$, where $S(\Phi_1) = 0.0921$, $S(\Phi_2) = 0.1513$, and $S(\Phi_3) = 0.0721$.

**Step 4:** Since, $S(\Phi_2) > S(\Phi_1) > S(\Phi_3)$, therefore, the RO of alternatives $\Xi_1, \Xi_2$, and $\Xi_3$ is "$\Xi_2 \succ \Xi_1 \succ \Xi_3$". Hence, $\Xi_2$ is the best alternative.

Table 2 and Figure 3 present a comparison of the RO of the alternatives $\Xi_1, \Xi_2$, and $\Xi_3$ obtained using different MADM approaches for Example 3. It is clear from Table 2 and Figure 3 that Garg and Kumar [7] MADM approach obtains the RO "$\Xi_1 = \Xi_2 = \Xi_3$", where it cannot distinguish RO between the alternatives $\Xi_1, \Xi_2$, and $\Xi_3$. While both Zou et al. [34] MADM approach and the proposed MADM approach obtain the same RO "$\Xi_2 \succ \Xi_1 \succ \Xi_3$". Thus, the proposed MADM approach effectively addresses and overcomes the shortcomings of Garg and Kumar [7] MADM approach.

**Table 2** The ROs of the alternatives obtained by different MADM approaches for Example 3

| MADM Approaches | ROs |
|---|---|
| Garg and Kumar [7] MADM approach | $\Xi_1 = \Xi_2 = \Xi_2$ |
| Zou et al. [34] MADM approach | $\Xi_2 \succ \Xi_1 \succ \Xi_3$ |
| Proposed MADM approach | $\Xi_2 \succ \Xi_1 \succ \Xi_3$ |


**Figure 3** Graphical comparison of POs obtained by various MADM methods for Example 3.

**Example 4.** Consider three alternatives $\Xi_1, \Xi_2$, and $\Xi_3$, and three attributes $\Lambda_1, \Lambda_2$, and $\Lambda_3$ with corresponding weights $\varphi_1 = 0.3, \varphi_2 = 0.4$, and $\varphi_3 = 0.3$. The decision maker wants to assess the alternatives with respect to the attributes under an IFS environment by using an IFN $\widetilde{\Phi}_{st} = \langle \widetilde{\eta}_{st}, \widetilde{\upsilon}_{st} \rangle$ to form the decision matrix $\tilde{D} = (\widetilde{\Phi}_{st})_{m \times n}$, given as follows:

$$\tilde{D} = \begin{array}{c} \\ \Xi_1 \\ \Xi_2 \\ \Xi_3 \end{array} \begin{array}{c} \begin{array}{ccc} \quad\Lambda_1 & \qquad\quad\Lambda_2 & \qquad\quad\Lambda_3 \end{array} \\ \begin{pmatrix} \langle 0.95, 0.01 \rangle & \langle 0.7, 0.01 \rangle & \langle 0.85, 0.002 \rangle \\ \langle 0.85, 0 \rangle & \langle 0.7, 0.02 \rangle & \langle 0.95, 0.004 \rangle \\ \langle 0.5, 0.2 \rangle & \langle 0.2, 0.5 \rangle & \langle 0.6, 0.3 \rangle \end{pmatrix} \end{array}.$$

In order to solve this MADM problem, we utilize the proposed MADM approach described in this paper as outlined below:

**Step 1:** As all the attributes are benefit-type attributes, normalizing the decision matrix is not required.

**Step 2:** Using the proposed AIFWG AO defined in Equation (8), we obtain the overall aggregated IFN $\Phi_s = \langle \eta_s, \upsilon_s \rangle$ of the alternative $\Xi_s$, where $\Phi_1 = \langle 0.8133, 0.0076 \rangle$, $\Phi_2 = \langle 0.8133, 0.0092 \rangle$, and $\Phi_3 = \langle 0.3674, 0.3629 \rangle$.

**Step 3:** Using Equation (9), we compute the score value $S(\Phi_s)$ corresponding to the alternative $\Xi_s$, where $S(\Phi_1) = 0.8725$, $S(\Phi_2) = 0.8719$, and $S(\Phi_3) = 0.4246$.

**Step 4:** Since, $S(\Phi_1) > S(\Phi_2) > S(\Phi_3)$, therefore, the RO of alternatives $\Xi_1, \Xi_2$, and $\Xi_3$ is "$\Xi_1 \succ \Xi_2 \succ \Xi_3$". Hence, $\Xi_1$ is the best alternative.

Table 3 and Figure 4 present a comparison of the RO of the alternatives $\Xi_1, \Xi_2$, and $\Xi_3$ obtained using different MADM approaches for Example 4. It is clear from Tables 3 and 4 that Zou et al. [34] MADM approach obtains the RO "$\Xi_1 = \Xi_2 \succ \Xi_3$", where it cannot distinguish RO between the alternatives $\Xi_1$ and $\Xi_2$. While Garg and Kumar [7] MADM approach and the proposed MADM approach obtain the same ranking "$\Xi_1 \succ \Xi_2 \succ \Xi_3$". Thus, the proposed MADM approach effectively addresses and overcomes the shortcomings of the Zou et al. [34] method.

**Table 3** The ROs of the alternatives obtained by different MADM approaches for Example 4

| MADM Approaches | ROs |
|---|---|
| Garg and Kumar [7] MADM approach | $\Xi_1 \succ \Xi_2 \succ \Xi_3$ |
| Zou et al. [34] MADM approach | $\Xi_1 = \Xi_2 \succ \Xi_3$ |
| Proposed MADM approach | $\Xi_1 \succ \Xi_2 \succ \Xi_3$ |

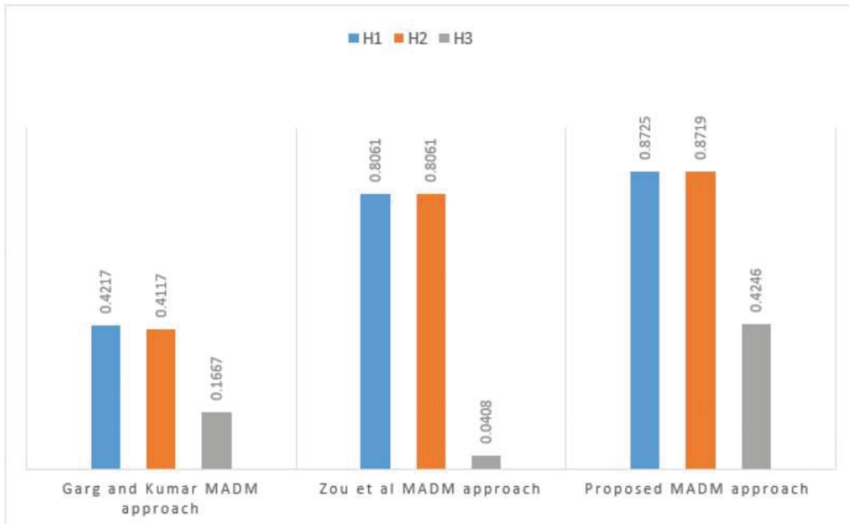


**Figure 4** Graphical comparison of POs obtained by various MADM methods for Example 4.

## 6 Conclusion

In this paper, we have introduced new operational laws for intuitionistic fuzzy numbers (IFNs) along with an advanced intuitionistic fuzzy weighted geometric (AIFWG) aggregation operator (AO). The desirable properties of the proposed AIFWG AO have also been presented to establish its validity. Based on the AIFWG AO, a new multi-attribute decision-making (MADM) approach within the IFNs framework has been developed. To showcase the advantages and validate the proposed MADM approach, three numerical MADM examples have been solved. The results of Examples 2, 3 and 4 clearly demonstrate that the proposed MADM approach is robust and highly effective. It effectively addresses the drawbacks found in existing MADM approaches developed by Garg and Kumar [7] and Zou et al. [34], where they cannot distinguish the ranking orders of the alternatives. Although the proposed MADM approach is effective but it has certain limitations. First, we are assigning weights directly to attributes, which may introduce bias and reduce the reliability of the results. Objective weighting methods, like CRITIC, MEREC, or entropy can be used to obtain weights of attributes and ensure consistency. Second, our proposed approach is limited to individual decision making, whereas real-life scenarios require group decision-making to incorporate the diverse opinions of multiple experts. Third, the proposed approach rely only on proposed AIFWG AO without incorporating classical MADM techniques such as EDAS, TOPSIS, VIKOR, MABAC, or TAOV, which could make the approach better. In the future, we aim to extend this work by developing group decision-making approaches using the proposed AIFWG AO within the context of IFNs, Pythagorean fuzzy numbers and q-rung orthopair fuzzy numbers. Furthermore, we intend to apply the proposed approach to real-life decision-making problems, such as pattern recognition, waste management, supply chain management, optimal site evaluation, financial risk assessment, and renewable energy project evaluation.

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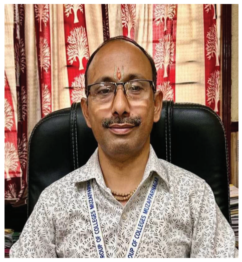
## Biographies

**Devraj Singh** is working towards his doctoral degree at Shri Venkateshwara University, Gajraula, U.P., India since 2022. He has obtained his Master degree in Mathematics during 2012–2014. He has more than one year of teaching experience in different subjects of Engineering Mathematics. His current research interests are on Aggregation operation, multi criteria decision making, Uncertainty theory, and intuitionistic fuzzy set theory. He has published 02 international articles in the different reputed journals.



**Vinit Kumar Sharma** is working as Dean Academics and Professor of mathematics in Shri Ram College, Muzaffarnagar and also working as visiting faculty & Research Supervisor, Faculty of Applied Sciences, Shri Venkateshwara University, Gajraula, U.P., India. 04 Ph.D. degrees have been awarded under his supervision and 04 Ph.D. researches are in progress. More than 32 research papers in reputed journals and 02 books have been published by him and also his 02 Patents have been granted. He has more than 23 years teaching and administrative experience in higher education. He is associated with many Educational and research organization as Guest faculty and member of some committees.

**Ashu Redhu** is working towards her Ph.D. degree with Amity School of Applied Sciences, Amity University Haryana, India. She has obtained her M.Sc. degree in mathematics from Maharishi Dayanand University, Haryana, India. She has also obtained her B.Sc. (Hons.) degree in mathematics from University of Delhi, India. She has more than one year of teaching experience in the different areas of Mathematics. Her research interest is in the field of fuzzy decision making, soft computing, Artificial Intelligence, and aggregation operators. Application areas include a wide range of industrial and structural engineering design problems. She has published 5 research papers in International Journals.