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Performance Analysis of Different Adaptive Algorithms for Equalization

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Abstract
The major problems in wireless communication are time dispersion and inter symbol interference. In order to cancel out the effect introduced by the unknown channel and to recover the original signal as from the distorted signal, a channel equalizer is required to compensate the effect of channel distortion, time variation and can adapt it-self to the changes in channel characteristics. The equalizers are expected to have fast convergence rate in communication systems which is difficult to achieve with conventional adaptive algorithms. LMS is widely used because it is simple and robust, but performs poor in terms of convergence rate. NLMS is an improved version of LMS and provides better convergence. RLS exhibit best performance but complex and unstable. In this paper we simulated adaptive algorithms such as LMS, NLMs and RLS algorithms in MATLAB and compared their performance.

Keywords- LMS, NLMS, RLS, Adaptive filtering, Convergence rate.

1. Introduction
From past few years, designing of adaptive filter has been a dynamic area of study and realization. An adaptive filter has the quality that automatically adjusts its parameter according to an adaptive algorithm (Haykin, 2008). Researchers has been studied Adaptive filtering algorithm for many decades because of their usefulness in diverse applications and equalization is one of them. In the field of digital signal processing there has been continuous progress in adaptive algorithm applications and several issues are found to be a focus for everyone's interest together with a large amount of calculation and the difficulty to attain the high-speed in real-time (Haykin and Moher, 2007). Figure 1 illustrates the basic building block of adaptive filtering process.

In this paper, to explain the performance of various adaptive algorithms, we took adaptive equalization application.

2. Channel Equalization
When the transmitted signal dispersed in time and amplitude, we need an equalizer to neutralize those effects. An adaptive equalizer is basically a linear adaptive filter that tries to make the replica of the opposite of transfer function of the channel that signal passes through. This process is also known as De-convolution often in digital signal processing (Barry et al., 2004). Figure 2 illustrates the basic process of an equalizer. Here the input u(t) is passing through the

unknown channel H(z). The equalization succession changes the impulse response of the unknown channel as $H^{-1}(z)$. The adaptive filtering algorithm calculates the error e(t) which is measured as the variation between the desired signal (delayed version of input signal) and filter output.

There is another Figure 3 which shows the transmission system with channel equalizer. Additive noise is here represented by the I(n) mixes with the input signal coming from the channel. In digital communication the distortion introduced by the channel is mostly recognized as pulse dispersion effect, which results in inter symbol interference. The receiver performance degrades with the additive noise. Equalizer eliminates the ISI and reduces noise to negligible amount (Razzak, 2015).

Mathematically the relationship between the unknown channel's transfer function h(n) and transfer function of adaptive filter as:

$$h1(n)h2(n)A = 1 \tag{1}$$

The implementation of an equalizer is generally done in the transversal filter's form. Initial training of an equalizer requires the knowledge of transmitted data symbol or we can say a delayed replica of them. Since they will be used as signal samples for adaptation of equalizer tap weight. Now, equalizer will try to obtain an output to be same as transmitted data signal. At the end of the training mode, equalizer would have converged it tap weight to an optimal value.

3. Adaptive Algorithms

3.1 Least Mean Square Algorithm

LMS algorithm adjusts the filter coefficients in such a way that produced the least mean squares of the error signal. It is based on stochastic gradient descent method that uses the gradient vector of the filter tap weight to converge of the optimal Wiener solution. The main advantage of the LMS algorithm is simple computation, easy implementation, fair convergence. The following mathematical model describes the LMS algorithm. The LMS algorithm updates is weight according to the method of steepest-descent algorithm that depends clearly on the information of the input and desired response signals but unlike steepest-descent LMS does not require exact measure of gradient vector but it uses a rough approximation of gradient (Reddy and Krishna, 2013).

The error of output filter can be expressed as

$$E(n)=d(n)-W^{T}x(n) \tag{2}$$

Approximation gradient can be expressed as

$$\zeta=-2E(n)x(n) \tag{3}$$

After substituting these expression, weight updation equation becomes

$$W(n+1) = W(n)+2\mu E(n)x(n) \tag{4}$$

Where W(n) is the weight value vector, x(n) is the input signal vector, μ is step-size which determines the convergence speed and e(n) is the error vector. For an N-tap filter, the number of operation get reduce to 2*N multiplication and N addition per Coefficient update. $E^2(n)$ is the mean square error, the difference between the output y(n) and the reference signal which is given by,

$$E^2(n)=[d(n)-y(n)]^2 \tag{5}$$

In order to attain a fast convergence speed and to keep a fast tracking ability in the steady state, large value for step size is taken. Large step size causes large steady misadjustment error. Figure 4 shows the flow chart of LMS algorithm.

3.2 Recursive Least Square Algorithm

There is a least square method upon which RLS is based. The RLS algorithm recursively find out the adaptive filter parameters that minimize the error. In the derivation of the RLS, the input signals are considered deterministic, while for the LMS and NLMS the input signals are considered stochastic (Tato and Miranda, 2002). In the absence of numerical measure of input signal, it is difficult to adjust the parameter of a filter but RLS can do that (Borisagar and Kulkarni, 2010). RLS algorithm has the similar procedures as LMS and NLMS except that it provides high convergence rate for fast fading channel, moreover RLS have some stability issues because of covariance updation formula, which is used for automatics change according to the estimated error as follows:

$$W^H(0)=0 \tag{6}$$
$$R(0)=\alpha^{-1}I \tag{7}$$
$$\nu(n)=R(n-1)x(n) \tag{8}$$

And gain vector can be expressed as

$$k(n)=\frac{\nu(n)}{\Delta+x^H(n)\nu(n)}$$

$$E(n)=d(n)-W^H(n-1)x(n) \tag{10}$$

$W(n)=W(n-1)+k(n)e(n)$ (11)

$R(n)=\Delta^{-1}R(n-1)+\Delta^{-1}k(n)x^{H}(n)R(n-1$ (12)

Where W(n-1) is the current weight value vector, W(n) is the next weight value vector, x(n) is the transmitted signal vector for adaptive filter and E(n) is the priori estimation error (Eleftheriou and Falconer, 1986). The inverse correlation matrix has to recalculated after the weight updation and process resume with new input values. Figure 5 shows the flow chart of RLS algorithm.

3.3 Normalized Least Mean Algorithm

LMS algorithm have a fixed step size for every iteration that is one of its disadvantage that can be overcome by using NLMS. The NLMS algorithm is an augmentation of LMS algorithm which uses the greatest step size. The flow chart is shown in Figure 6. It can be said NLMS is an extension of LMS algorithm (Ifeachor and Jervis, 2002). In case of noise cancellation there are many factors which affects the performance of system such as input signal power and its amplitude. The step-size for NLMS calculated by the following formula as:

$\text{Step-size}=1/u(n)u^{t}(n)$ (13)

Where
u(n)=input vector

Implementation of NLMS is very much similar to LMS. The steps required for weight updation for NLMS is given as:

$Y(n)=w^{t}(n)\, u(n)$ (14)

$E(n)=d(n)- y(n)$ (15)

$\mu=1/u^{t}(n)\, u(n)$ (16)

$c(n+1)=c(n)+\mu e(n)\, u\, (n)$ (17)

4. Simulation Results

The simulation is done in MATLAB. MATLAB is an interactive environment for numerical computation, design and problem solving. MATLAB is used for the variety of applications including signal processing, communication, control system and computational biology. The MATLAB simulation can be performed either by using MATLAB M-file or by Simulink interface. The experimental work here is performed using MATLAB M-file.

4.1 Specification of LMS algorithm in MATLAB

a. If Number of filter taps =30 and Step size(μ)=0.01, Then the result is shown in Figure 7.

Specification of RLS algorithm in MATLAB

b. If Forgetting factor (λ)=0.98 and Regularization factor(α)=1, then result shown in Figure 8.

4.2 Specification of NLMS algorithm in MATLAB

c. If Number of filter taps=30 and Step size(μ)=0.02, then result is shown in Figure 9.

5. Conclusion

In this paper the comparison of adaptive algorithms is performed on the basis of convergence rate and simulation is done using the software tool MATLAB. The simulation results show that RLS provides better convergence rate but at the cost of high computational complexity and less stability. LMS filtering is relatively good and it is simple and easy to realize hardware but provides slower convergence rate. Convergence speed can be improved by choosing an appropriate value of step-size but the drawback of LMS algorithm is having a fixed step-size parameter for each iteration, which can be overcome by NLMS which uses the greatest step-size and also provides better convergence speed than LMS.

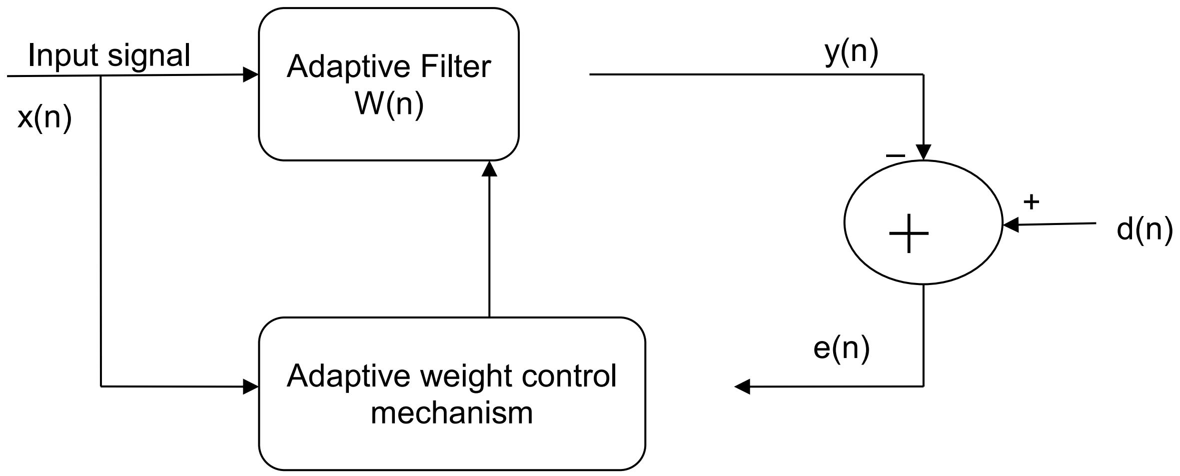

Figure 1. Block diagram of adaptive filtering

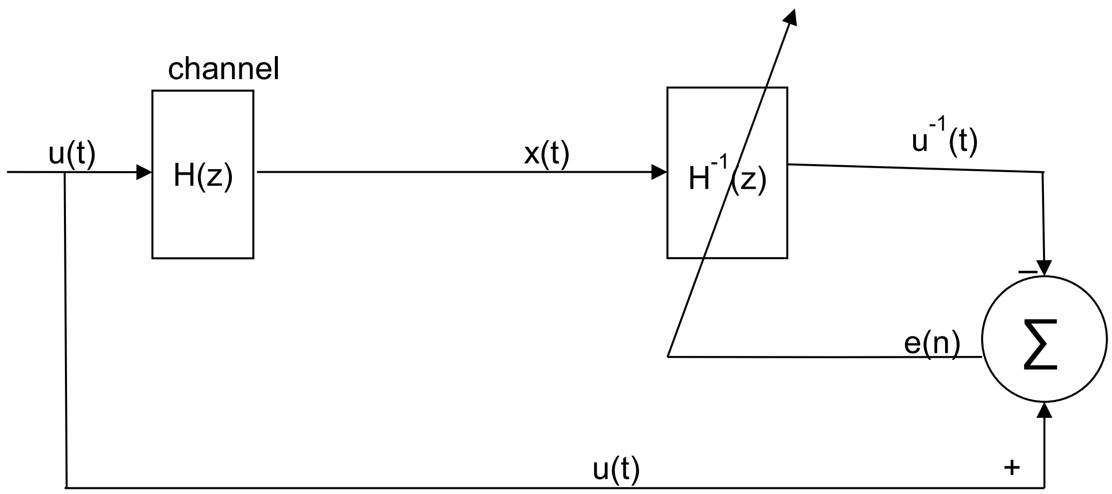

Figure 2. Basic blocks of an equalizer

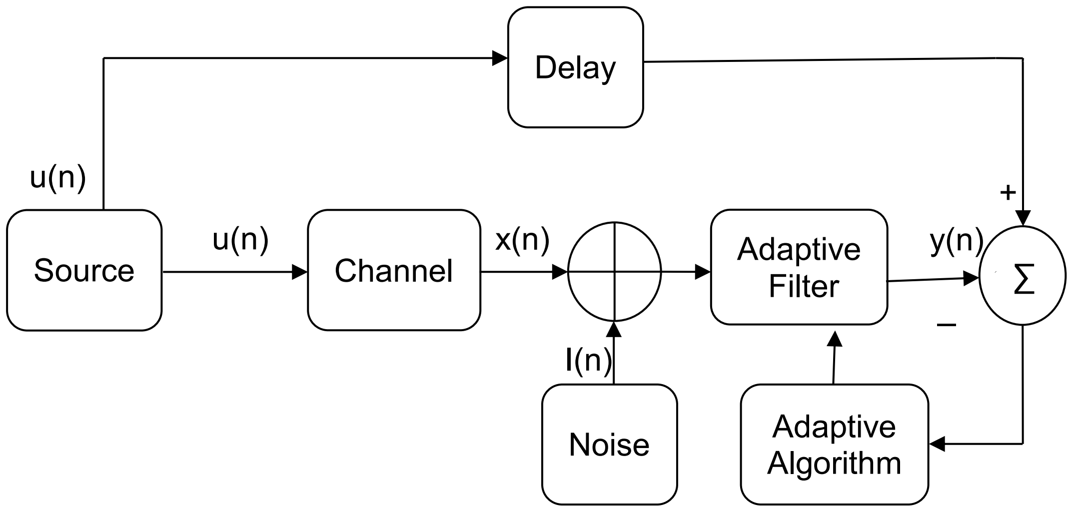

Figure 3. Transmission system with channel equalizer

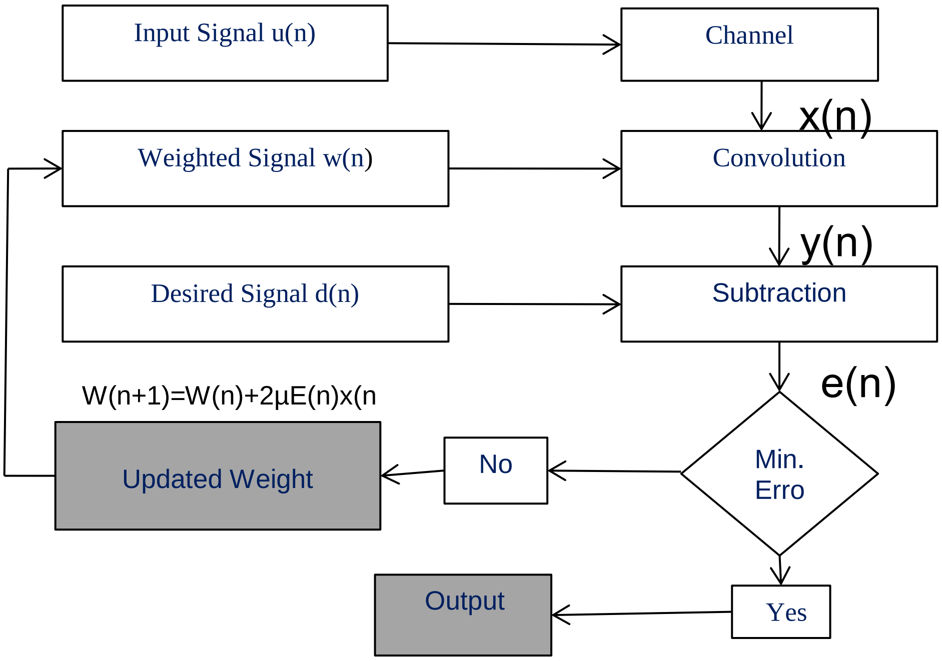

Figure 4. Flow chart of LMS algorithm

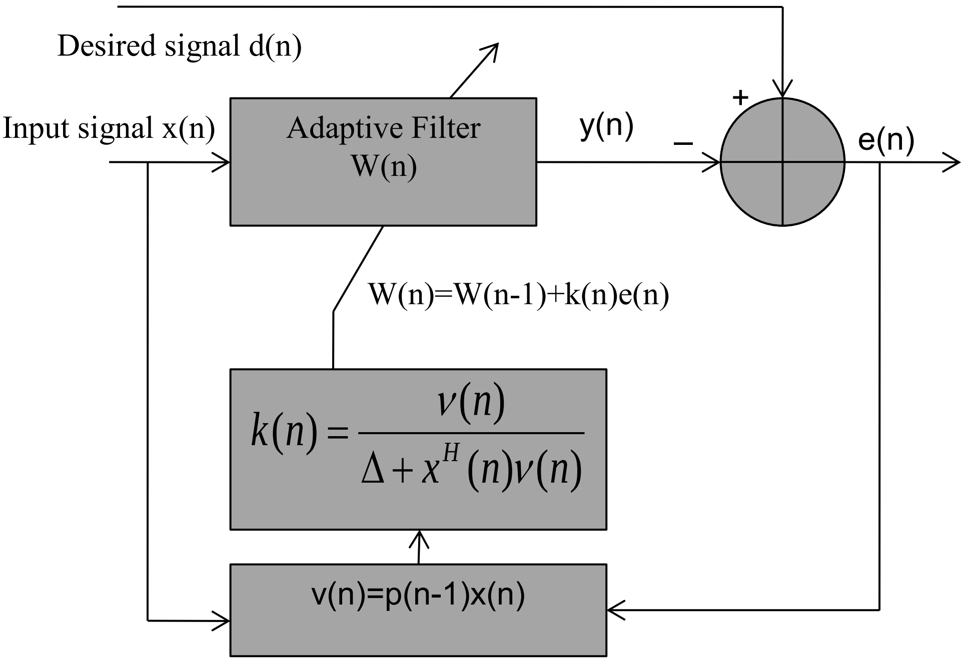

Figure 5. Flow Chart of RLS Algorithm

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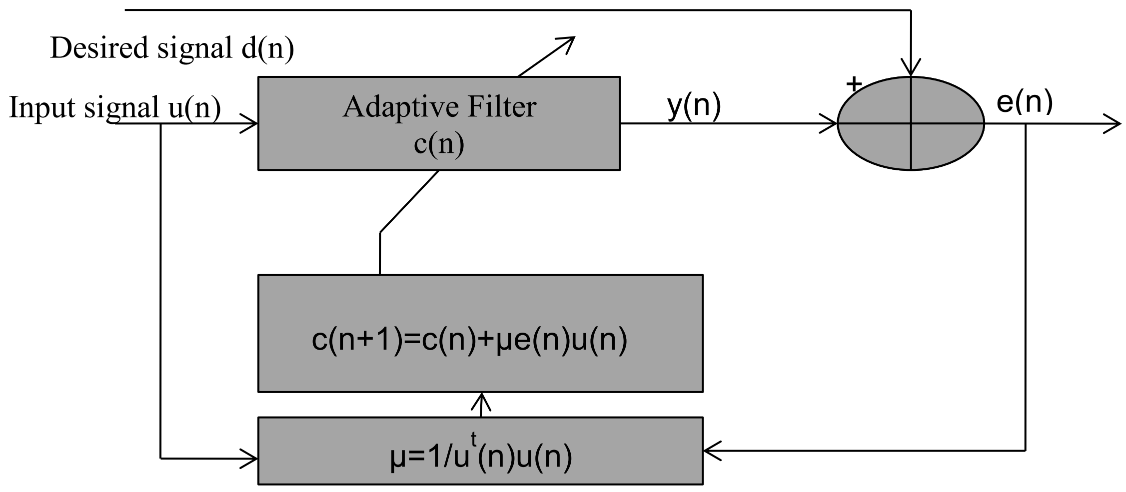

Figure 6. Flow Chart of NLMS Algorithm

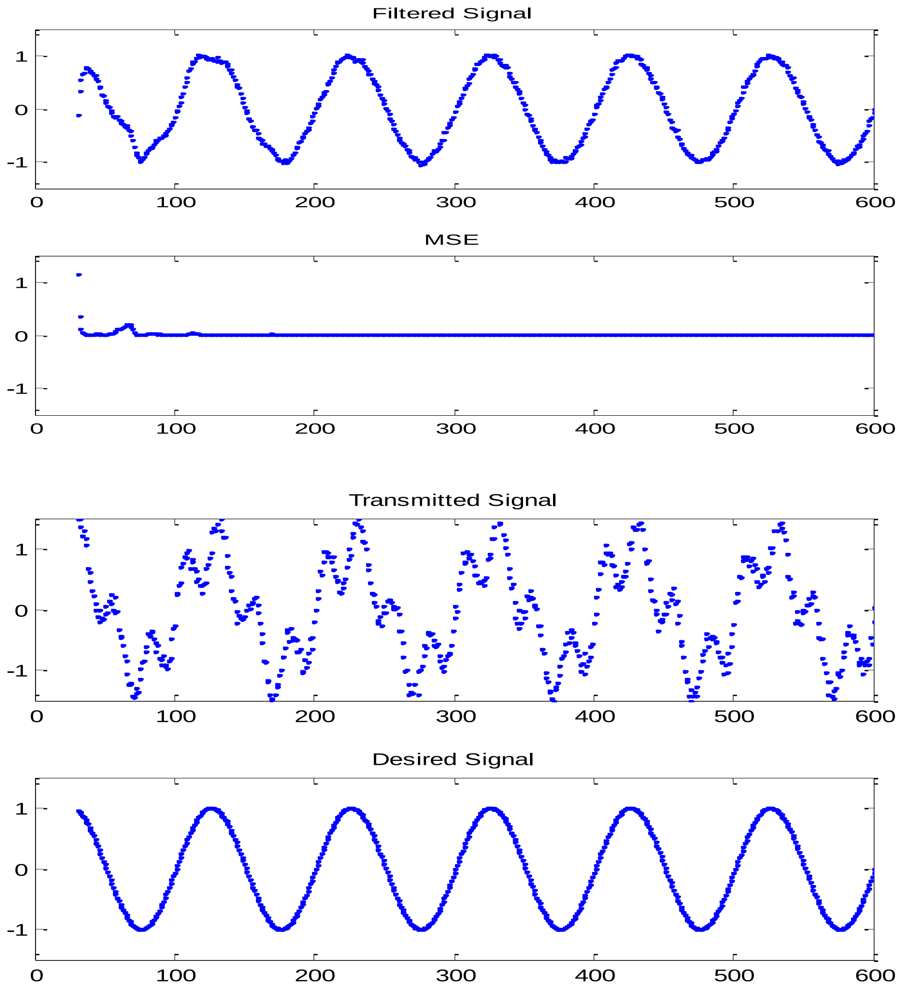

Figure 7. Simulation result of LMS algorithm

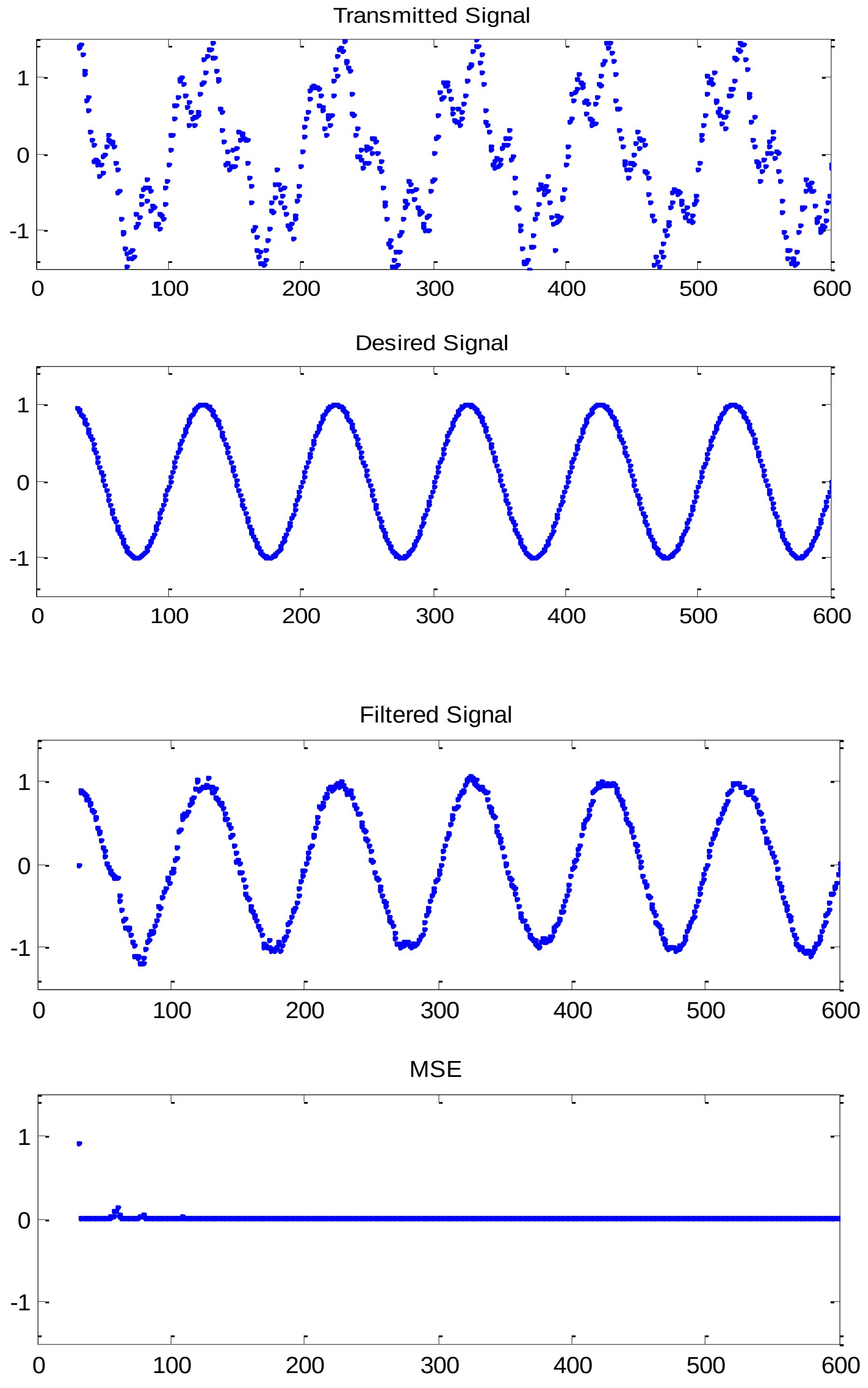

Figure 8. Simulation result of RLS algorithm

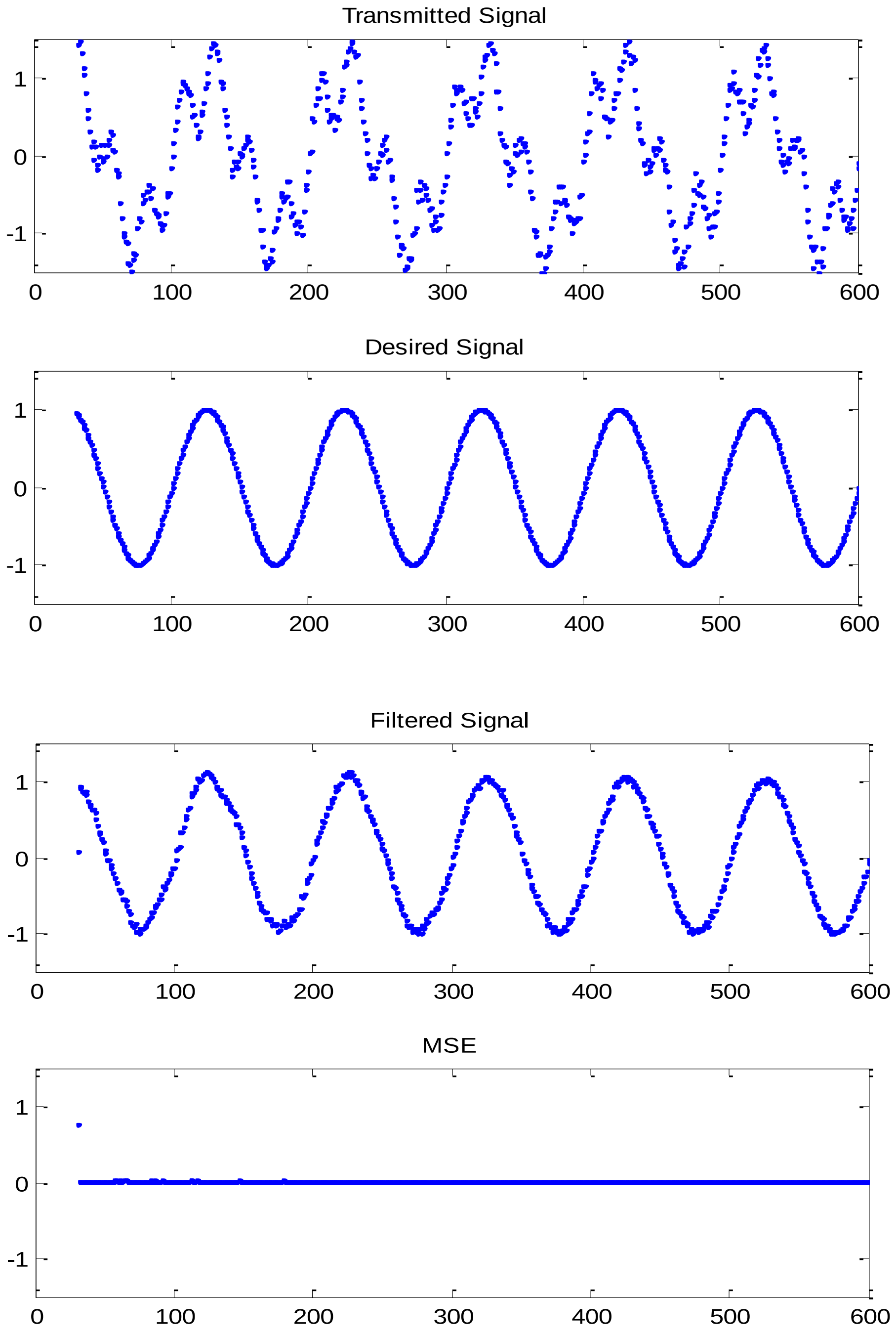

Figure 9. Simulation result of NLMS algorithm

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